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Information and Competition in U.S. Forest Service Timber Auctions

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Abstract

This paper studies the bidding behavior of firms in U.S. Forest Service timber auctions in 1976–1990. When conducting timber auctions, the Forest Service publicly announces its estimates of the tract characteristics before the auction, and each bidder additionally has an opportunity to inspect the tract and form its own private estimates. We build a model that incorporates both differential information and the fact that bids placed in timber auctions are multidimensional. The theory predicts that bidders will strategically distort their bids based on their private information, a practice known as "skewed bidding." Using a dataset that includes both the public ex ante Forest Service estimates and the ex post realizations of the tract characteristics, we test our model and provide evidence that bidders do possess private information. Our results suggest that private information affects Forest Service revenue and creates allocational inefficiency. Finally, we establish that risk aversion plays an important role in bidding behavior.

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1 Introduction

This paper studies the extent to which bidders in auctions obtain and strategically exploit private information. Since the private signals of bidders are not typically observable by an econometrician, it is in general difficult to test theories about how bidders make use of these signals. In this paper we identify a setting in which *ex post* information about attributes of the object is available, attributes that affect the value of the object to all bidders. Using this information, we are able to empirically examine the extent to which bidders possess private information about these common attributes, as well as the effect of such private information on behavior and the outcome of the auction. In particular, we are able to test predictions in the spirit of Milgrom and Weber's (1982) model of the "mineral rights" auction.

The setting for our study is the U.S. Forest Service timber auction program in the Pacific Northwest from 1976 to 1990.[1] Our analysis relies on a particular auction format — called a "scale sale" — used by the Forest Service. Each tract of timber contains multiple species. Before the sale, the Forest Service conducts a "cruise" and estimates the volume of each species (as well as other factors such as timber quality and logging costs). These estimates are publicly announced at least a month before the sale, at which time bidders have the opportunity to hire their own cruisers and form independent estimates. In a scale sale, firms bid a price per unit of volume for each species. The winner is the firm that places the highest estimated total bid, computed as the sum of the prices on each species weighted by the Forest Service quantity estimates. The contract requires the winner to remove *all* designated timber from the tract. As the timber is removed, the Forest Service measures the volume of each species, and the winner pays for the timber removed at the rates specified in the bid. Thus, there may be a significant gap between the average bid, weighted by the Forest Service estimates, and the average payment, weighted by the actual proportion of each species. Such a gap is typical: on tracts with two main species of timber, the Forest Service's estimate of the proportion of timber that is the primary species turns out to be within 5% of the actual proportion that is removed on only half of the sales.

While scale sales may appear particular to the Forest Service, similar rules are in fact used in many government auctions. Examples include bidding for highway construction contracts and "indefinite quantity" procurement auctions for goods and services. In such procurements, the government specifies its estimated demand for different goods, which are used as weights in computing the total bids. But, the government can also choose to order more than those estimated quantities.

Despite the prevalent use of scale sales, the *equilibrium* behavior of firms in such auctions has not to our knowledge received a general theoretical or empirical analysis in the economics literature.

[1] Other recent analyses of Forest Service timber auctions include Hansen (1986), Leffler and Rucker (1988), Cummins (1995), Baldwin, Marshall, and Richard (1997) and Haile (1998).

Our formal model of equilibrium bidding behavior in scale sales can be described as follows. There are two species, 1 and 2, and each (risk-averse) bidder receives a signal about the true proportion of species 1, and places bids for each species, b_1 and b_2. If the Forest Service has estimated that the proportion of species 1 timber is x_1, and the total quantity is Q_{EST}, then the total bid B is computed as $B = Q_{EST} \times (b_1 x_1 + b_2(1 - x_1))$. The winner, who submitted the highest total bid, ultimately pays $P = Q_{ACT} \times (b_1 \rho_1 + b_2(1 - \rho_1))$, where Q_{ACT} is the realized quantity on the tract, and ρ_1 is the realized proportion of species 1 on the tract. In such an auction, bidders have an incentive to "skew" their bids onto the species they believe the Forest Service has over-estimated. The intuition is that a bidder can achieve the same total bid (and thus the same probability of winning the auction) while paying less if the total bid is allocated mainly to species that are over-estimated, since each dollar bid on an over-estimated species increases the total bid by more than it increases the expected payment.

To see how this would work in practice, consider a simple example. Suppose that the Forest Service announces a tract with 1000 units of volume, and estimates that half of the tract is Douglas Fir, and half is Western Hemlock. A bidder can achieve a total bid of $100,000 by bidding $100 on each species. Another way to achieve the same total bid is by bidding $150 per unit on Douglas Fir, and $50 on Spruce. Suppose that a bidder's cruise indicates that the expected proportion of Douglas Fir is in fact 0.25, not 0.5 as the Forest Service estimated. Then the bidding strategy ($100,$100) yields an expected payment of $100,000. However, the bidding strategy ($150,$50) yields an expected payment of $75,000.

We analyze the equilibria of sealed bid auctions and a stylized oral ascending bid auction. The theory exploits a basic insight, that bid decisions can be decomposed into two components: a choice of what total bid B to submit, and a decision about how to allocate that bid across the two species, that is what level of bid skew, $\Delta b = b_1 - b_2$, to select. We show that bidders who have higher signals about the extent to which the Forest Service has over-estimated one species (i.e. about $|x_1 - \rho_1|$) will skew their bids more aggressively, and will be more optimistic about the gap between submitted bid and payment. As a consequence, they will also submit higher bids. The oral ascending bid auction works in a similar fashion, although some subtleties arise from the fact that bids at any point in time have two dimensions, a total bid and a skew. The theory we develop has the attractive feature that skewing behavior at each stage of the auction corresponds to the optimal skew of a bidder who is just indifferent about dropping out of the auction. Thus, we can give a precise interpretation to losing bids in our oral auction data.

We test our theoretical predictions using data from sales in Oregon and Washington during 1976–1990. We match bidding data, which includes information from the Forest Service cruise as well as the bids and bidder identities, with *ex post* cutting data from the tract. Since we observe both the estimated quantities of each species and the quantities that are removed, we are able to

use the difference between the estimated and actual proportions of species 1, $x_1 - \rho_1$, as a proxy for the private information potentially available to the firms. We refer to this difference, $x_1 - \rho_1$, as the "mis-estimate" of the Forest Service. Using this data, we estimate the extent to which information about Forest Service mis-estimates affects bidding behavior and Forest Service revenue.

Our empirical analysis broadly confirms the predictions of our theoretical model. We find strong evidence that bidders respond to mis-estimates by skewing their bids, and also that higher-ranked bids tend to be better matched to mis-estimates than lower-ranked bids. Likewise, we see that conditional on the level of a bidder's total bid, the amount paid (or the amount the bidder would have paid, had they won) is decreasing in $|x_1 - \rho_1|$. Forest Service revenue is also systematically affected by mis-estimates. We further find that in a given sale, there is often heterogeneity in the extent and direction of the skew, consistent with private information on the part of the bidders. Higher bidders at an auction tend to also skew their bids more, and we find some evidence that bidders incorporate information in the course of an oral auction. Finally, we frequently observe firms using moderate skews, which can only be optimal for risk averse bidders (in or out of equilibrium).

The results in this paper contribute to the existing literature in several ways. We have extended Milgrom and Weber's (1982) theory of mineral rights auctions to the case of "scale sales," as described above. Previous papers have looked at auctions with multidimensional bids and scoring rules, but these studies consider settings with independent private values, and there is no analog to the skewing behavior identified here.[2] Baldwin (1995) and Wood (1989) have formally analyzed skewing: Baldwin considers the bid allocation problem, taking the bid level as exogenous. Wood restricts attention to pre-selected linear bidding rules and looks at second-price auctions. Neither considers both the full decision problem and equilibrium behavior when information is dispersed among the bidders. Baldwin also empirically estimates the degree of risk aversion of bidders at Forest Service timber auctions. Her dataset, however, does not include the ex post realizations of the volumes of each species on the tract, and thus her conclusions are based on revealed preference rather than direct measures of the returns to skewing.

The effects of private information in "mineral rights" auctions have also been empirically analyzed by Hendricks and Porter (1988) in the context of leases for off-shore oil drilling. Hendricks and Porter identify the effects of informational advantages using ex post information about the tracts (in this case, the amount of oil in the tract), and testing to see whether "neighboring" firms have better information. In contrast to oil lease auctions, scale sales have the attractive feature that it is possible to distinguish the effect of information on each bidder's decision-theoretic choice about skewing (which does not directly affect the probability of winning the auction) from her

[2] Osband and Reichelstein (1985), Che (1993) and Bushnell and Oren (1994) study auctions where bidders can bid in two dimensions (cost and quality in the case of Che, and fixed and variable cost in the case of Bushnell and Oren). The winner is determined by a scoring rule.

strategic decision about the total bid.

This paper is organized as follows. Section 2 provides background on the Forest Service Timber Program and describes some of the relevant institutional details of the timber auctions. Section 3 presents a formal model that captures many of the main elements of bidding behavior at scale sales. Section 4 describes the data, while Sections 5 and 6 provide our main empirical analysis. Section 7 concludes.

2 Background: The Forest Service Timber Program

In the northern and western regions of the U.S., the national forests have been the primary source of timber for mills, logging companies, and forest products companies. During 1976–1990, the Forest Service conducted well over a thousand auctions per year in these areas, generating annual revenue of around $1 billion. Our empirical work will focus on Forest Service Regions 5 (California) and 6 (Oregon and Washington); in the 1980s, these regions accounted for two-thirds of all Forest Service timber sold and 80% of all Forest Service timber receipts.

USFS Timber Auctions, 1976(1)–1990(2), Statistics

	Region 5	**Region 6**
Sales	6009	16,857
Avg. Volume (mbf)[3]	3752.6	3994.6
Avg. Reserve Price ($)	184,897	329,280
Avg. Winning Bid ($)	542,047	682,227
Avg. Bid per unit vol ($/mbf)	110.34	151.69
Avg. Number of Bidders	3.67	5.07
% Oral Auctions	47.5	87.1

Since details of the Forest Service timber program have been discussed elsewhere (see, for example, Baldwin, Marshall, and Richard (1997)), we touch only on a few key aspects of the industry organization, and focus our discussion on the process through which bidders acquire information and prepare a scale sale bid.

Bidding in Forest Service sales is undertaken by a diverse collection of timber conglomerates, smaller mills, and independent logging operations. Timber from a sale is frequently processed in mills proximate to the national forest. In Region 6, there are several hundred mills of different types, each demanding different species and qualities of timber. Conglomerate firms (such as Weyerhauser, Boise-Cascade, and Georgia-Pacific) own many mills all over the country, and may be able to process the whole range of species and timber qualities on a given tract. Smaller mills

and independent "gyppo" logging operations do not have this ability. When these smaller bidders win Forest Service auctions, they tend to resell some or all of the timber. In the 1980s, this resale was done either through private bilateral trades or, in smaller quantities, through prices posted by the mills (see Haile (1998) for an analysis of resale in timber auctions).

The Forest Service begins preparation for a sale several months prior to the sale date. The forest manager organizes a cruise and publicly announces the findings at least 30 days before the sale date. The manager also decides, based on tract characteristics and expected competition, whether to conduct the sale by oral or sealed bidding — in Region 6, the great majority of the sales are oral auctions. Once the sale is announced, each firm must "qualify" for an auction by submitting a deposit of 10% of the bid in a sealed bid auction, or 10% of the appraised value of the sale in an oral auction.[4] This deposit is held until the contract is awarded.

In addition to the scale sale format already mentioned, the Forest Service occasionally uses an alternative "lump-sum" format. In a lump-sum sale, each firm makes a fixed bid, and the firm with the highest bid wins the auction and pays that bid, irrespective of the realized volumes of each species. As described above, bidders in a scale sale submit a bid rate for each species and the winner pays the bid rate for the realized volume of each species. The reserve price mechanism differs slightly between the two formats: in a lump-sum sale, the Forest Service sets a fixed reserve price for the whole tract; in a scale sale, there is a minimum acceptable bid for each species.

Scale sales are used mainly in the Northern and Western regions of the country. The main motivation for using scale sales is to reduce risk on the part of the bidders.[5] If the bidders face substantial risk about the value of the sale, their bidding will be less aggressive (not only do they require a risk premium, but they may be more concerned about the winner's curse, whereby the bidder who wins is generally the one whose cruiser gave the most optimistic estimate). In lump-sum bidding, bidders bear all of the risk about the species composition of a tract. In contrast, a scale auction automatically mitigates uncertainty about the *total volume* of timber on a tract, and bidders can in principle insure against uncertainty concerning the *proportion of each species* on the tract by bidding the same profit margin on each species.

Before submitting their sealed bids or qualifying bids, the bidders have the opportunity to cruise the tract and form their own estimates of tract characteristics. Cruising a tract has traditionally been considered something of an art by industry insiders, and many cruisers have an undergraduate degree in forestry (such a degree as well as two years of experience are requirements for admission

[4]In oral sales, a bidder can choose to raise the bid above the reserve price in this initial qualifying round of bidding, in which case the deposit must be equal to 10% of the bid.

[5]Forest Service personnel also bear political costs when their ex ante estimates are incorrect. The scale sale system has the potential to reduce the sensitivity of bidder outcomes to volume mis-estimates, thus making it politically desirable. Firms in the industry have historically excercised great influence over Forest Service policy.

to the industry association). According to industry sources, beginning cruisers in the 1990s made about \$30,000 to \$40,000 per year, while more experienced cruisers made \$60,000 or more. Large forest product companies have in-house cruising staffs, while smaller companies may use for-hire cruisers from consulting companies. These for-hire cruisers typically price their services either by the acre or by the hour. While the costs vary substantially from tract to tract, one firm estimated a "typical" cost of \$10/acre. The average tract size in our sample is 380 acres, putting this cost at about .6% of the tract value. While bidders reportedly spend more resources on cruising for "lump sum" rather than "scale" sales, industry sources in the Pacific Northwest report that it is unusual for a bidder to place a bid on a tract without cruising. Moreover, firms that have incurred the costs of surveying a tract generally submit bids — thus, one can think of the decision to survey a tract as roughly equivalent to an entry decision.

To cruise a tract, the cruisers go out into the forest for several days. A difficult aspect of the job is detecting potential defects in trees. For example, insects can damage old growth trees, but this may be difficult to verify from external characteristics; and different kinds of insects may have caused problems in different areas of a forest. Cruisers must guess at the defects in trees and determine the "merchantable" timber, which will be sold at the prices bid at the auction, and the "non-merchantable" timber, which is essentially scrap.

The Forest Service has long been aware of the problems of "skewed bidding" in scale sale auctions (see, for example, GAO Report RCED-83-37, which documented revenue losses from skewing). In order to limit such behavior, rules have been adopted over time that limit the scope for skewing. For example, during the 1980s, in some forests bidders could only place bids above the reserve price on "major species," which account for more than 25% of the volume. In 1995, some parts of the Forest Service moved to "proportional bidding," whereby bidders are required to distribute their bids across species according to fixed proportions.

3 The Model

In this section, we build a stylized model of equilibrium bidding behavior in scale auctions. We abstract from a number of considerations, most notably heterogeneity in the values of firms for each species of timber, as well as privately observed cost or inventory shocks. Thus, our results can be viewed as providing insight into one component of bidding behavior, while stopping short of a full analysis of the real-world auction setting. In particular, with private values as well as a common component, the tract would be allocated in part on the basis of individual preferences and in part on the basis of signals about the common value. It is important to note, however, that if the FS had the same information as the bidders about the value and extraction costs of each species of

timber (as in our formal model), there would be no need for an auction at all.[6]

The model can be outlined as follows. There are two species, 1 and 2. These species have commonly known values $v_1 \geq v_2$, which are the same for all bidders. The auctioneer provides estimates of the total volume on the tract, Q_{EST}, and the proportion of the volume that is species 1, x_1 (and let $x_2 = 1 - x_1$). These estimates are announced, together with the reserve prices for each species, r_1 and r_2. We assume that $v_1 > r_1$ and $v_2 > r_2$.

Each bidder has an identical utility function defined over wealth $u(\cdot)$, which we assume to be strictly increasing and (weakly) concave. Let ρ_i indicate the true, unobserved, proportion of species i, and let $\delta_i \equiv x_i - \rho_i$ be the difference between the estimated and actual proportion, so that $\delta_i > 0$ corresponds to an over-estimate of the proportion of species i. All bidders observe a *public* signal, $\chi \in \{1, 2\}$, which is informative about the direction of the auctioneer's mis-estimate. We will assume that χ conveys enough information that all bidders will choose to skew in the same direction. This substantially simplifies the analysis, without disturbing the underlying ideas, by allowing us to ignore the extent to which bidders learn about the *direction* of the mis-estimate when they discover that they have won. Each bidder j also observes a *private* signal, d_χ^j, about the auctioneer's mis-estimate (under the assumption below, $E[\delta_\chi | d_\chi^j]$ will be increasing in d_χ^j). We assume that the total volume, Q_{ACT}, is fixed and commonly known to the bidders, though not necessarily equal to Q_{EST}. Alternatively, we can allow Q_{ACT} to be stochastic so long as it varies *independently* of δ_χ. We make the following assumptions about the random variables:

(A1) *(i)* support$(\delta_\chi | \chi, d_\chi^j) = [x_\chi - 1, x_\chi]$ and $E[\delta_\chi | \chi, d_\chi^1, ..., d_\chi^J] > 0$ for all $(d_\chi^1, ..., d_\chi^J)$; *(ii)* conditional on x_χ, $(d_\chi^1, ..., d_\chi^J, \delta_\chi)$, (χ, δ_χ) are strictly affiliated and $(d_\chi^1, ..., d_\chi^J)$ are exchangeable.

Part *(i)* implies that the public signal is sufficiently informative that the bidders always agree in their assessment of the expected *direction* of the mis-estimate; however, even after seeing the public signal, any mis-estimate is possible (that is, the support does not change with the public signal). The affiliation and exchangeability assumptions *(ii)* will allow us to establish the critical comparative statics results in the analysis.

We now consider the sealed bid auction and the oral ascending auctions in turn. We then discuss the empirical implications of the model. Proofs are deferred to the Appendix.

3.1 The Sealed Bid Auction

Consider a first-price, sealed bid auction. After observing the auctioneer's announcements, the public signal χ, and a private signal d_χ^j, each bidder j submits a sealed bid for each species, b_1^j

[6] For this reason, it is trivial to describe the *optimal* mechanism in this context: the Forest Service should simply set a rate for each species equal to its value and sell to any bidder. If one were to add a private value component, the question of optimal auction design would become interesting.

and b_2^j, to the auctioneer. The total bid is computed as $B^j = Q_{EST} \sum_i b_i^j x_i$ and the highest bid wins. After the sale, the true total volume Q_{ACT} and proportion of species 1, ρ_1, are realized. Ordering the bidders $j = 1, .., J$ in descending order of their total bids, the winning bidder pays $P^1 = Q_{ACT} \sum_i b_i^1 \rho_i$.

It is useful to think about each bidder's decision in two parts: selecting a total bid B and allocating that bid over the two species, i.e. choosing $\Delta b_\chi = b_\chi - b_{\neg\chi}$.[7] Given opponent strategies, each bidder j faces a two-part optimization: first, for any total bid B^j, what is the optimal bid allocation $\Delta b_\chi^j(B^j, d_\chi^j)$? Second, given that any total bid will be allocated optimally, what is the optimal total bid $B^j(d_\chi^j)$?

Only a bidder's decision about his total bid affects his probability of winning the auction; his bid allocation is relevant only if he wins. Thus, bidder j's bid allocation problem, for which we assume a unique solution, can be written as follows (letting $\Delta v_\chi \equiv v_\chi - v_{\neg\chi}$, and $V = Q_{EST}(\mathbf{v} \cdot \mathbf{x})$):

$$\max_{\Delta b_\chi} E_{\delta_\chi} \left[u \left(Q_{ACT} \cdot \left((\Delta b_\chi - \Delta v_\chi) \cdot \delta_\chi + \frac{V - B^j}{Q_{EST}} \right) \right) \mid d_\chi^j, \chi;\ \forall k \neq j,\ B^k(d_\chi^k) < B^j \right], \quad (1)$$
$$\textit{subject to:} \qquad b_1 \geq r_1,\ b_2 \geq r_2,$$

where j conditions on winning with total bid B^j (accounting for the winner's curse), as well as his own private information.

The bid allocation problem is formally equivalent to a portfolio problem; think of the bidder as choosing his "investment," $\Delta b_\chi - \Delta v_\chi$, in the risky asset, δ_χ. A bidder can eliminate the risk due to δ_χ by bidding a constant profit margin on each species (setting $\Delta b_\chi = \Delta v_\chi$). We refer to this as "full-insurance," and we refer to any departure from this strategy as a "skew." Assumption *(A1)* ensures that the expected value of the risky asset δ_χ is positive. Thus, if a bidder is risk-neutral, he will invest as much of his total bid B as possible on species χ, maximizing $\Delta b_\chi - \Delta v_\chi$ subject to the reserve price constraints. The latter finding is quite robust (and in particular, it does not depend on our assumption that χ is publicly observed). So long as the bidder believes that there is, on average, a mis-estimate, risk-neutrality implies that skewing should be maximal. In contrast, for a risk-averse bidder, the solution will involve some investment in the risky asset, that is, $\Delta b_\chi - \Delta v_\chi > 0$, but in general the solution will be interior.

Proposition 1 *Consider the bid allocation problem. (i) If a bidder is risk-neutral, then the optimal bid sets $b_{\neg\chi} = r_{\neg\chi}$. (ii) If the bidder is risk-averse, and $B^j \geq V - (v_2 - r_2)$, it is optimal to set $\Delta b_\chi^j - \Delta v_\chi > 0$.*[8]

[7] Note that $b_\chi = B/Q_{EST} + \Delta b_\chi (1 - x_\chi)$ and $b_{\neg\chi} = B/Q_{EST} - \Delta b_\chi x_\chi$, so choosing B and Δb_χ uniquely determines b_χ and $b_{\neg\chi}$ (and vice-versa).

[8] If $B^j < V - (v_2 - r_2)$, a risk-averse bidder will skew maximally, setting $b_{\neg\chi} = r_{\neg\chi}$, but the reserve price constraints imply that $\Delta b_\chi < \Delta v_\chi$.

Proposition 1 tells us that the solution to the bid allocation problem entails taking on some risk, despite the fact that a full-insurance policy is available. Consistent with portfolio theory, the next Proposition establishes that bidders with higher signals about Forest Service mis-estimates will skew their bids more aggressively.

Proposition 2 *Suppose that $B^k(d^k_\chi)$ is nondecreasing in d^k_χ for all $k \neq j$, and that $B^j \geq V$. If preferences satisfy CARA or IARA, any optimal $\Delta b^j_\chi(B^j, d^j_\chi)$ will be nondecreasing in B^j and d^j_χ.*

Proposition 2 also establishes that the optimal skew is increasing in the total bid. One reason for this is straightforward — when opponents use nondecreasing bidding functions, winning with a higher total bid means that the distribution of (defeated) opponent types is more favorable. There is also a further effect, which is the reason we require constant or increasing absolute risk aversion (CARA or IARA). When the total bid is higher, bidders evaluate the species composition risk from (essentially) a position of decreased wealth. Under decreasing absolute risk aversion (DARA), this decrease in wealth would lead to an increase in risk-aversion and a reduced propensity to skew, introducing a competing effect that, though presumably small, would greatly complicate the model. Thus, while DARA would seem to be a reasonable assumption (and indeed, one that is supported by Baldwin's (1995) empirical work), we rule it out.

We now establish the existence of a pure strategy Nash equilibrium in which bidders with more optimistic signals skew more aggressively and submit higher total bids. Our approach is to show that, for each bidder j, if all opponents $k \neq j$ use identical strategies $(\Delta b^k_\chi(B^k, d^k_\chi), B^k(d^k_\chi))$ that are nondecreasing in (B^k, d^k_χ) and d^k_χ respectively, then bidder j's best response must be similarly nondecreasing. Under this condition, Athey (1997) has shown that a pure strategy Nash equilibrium exists.

Proposition 3 *(i) For each bidder j, suppose that all opponents $k \neq j$ choose B^k according to the strategy $\beta(d^k_\chi)$, which is nondecreasing in d^k_χ with $\beta(\cdot) \geq V$. If preferences satisfy CARA or IARA, any best response function $B^j(d^j_\chi)$ will be nondecreasing and have $B^j(\cdot) \geq V$. (ii) Thus, a pure strategy Nash equilibrium exists in nondecreasing strategies where all bids submitted are greater than or equal to V.*

Recall from Proposition 1 that bidders choose to skew their bids as the optimal solution to their first-stage portfolio problem. Proposition 3 implies that bidders *must* skew to win. A bidder who chooses a full-insurance policy that does not result in a sure loss can bid at most $B = V$. He will thus be outbid by an opponent who believes that the Forest Service estimates are incorrect, since by skewing, such an opponent can generate a higher total bid, while still expecting to pay less than the full-insurance policy. The monotonicity of the equilibrium bid functions implies that the bidder who gets the highest signal about the mis-estimate will skew his bid the most and win the auction.

Our model assumes that bidders have identical signal quality and that this quality is exogenous and fixed. Hendricks and Porter (1988) have demonstrated the importance of superior information in oil lease auctions; bidders who have already drilled a neighboring tract are better informed than outsiders. In the case of timber auctions, this extreme form of information asymmetry seems unlikely, although it seems plausible that firms with greater resources could invest more in information gathering. Indeed, there is significant evidence that bidders can invest varying amounts in "cruising" the tract. Persico (1998) has shown that endogenizing the information acquisition decision can have important consequences for revenue predictions. Essentially all of the comparative statics results we will seek to test would persist in a symmetric model with endogenous information acquisition. Heterogeneity of information quality is a more difficult problem, and we will not pursue it here.

3.2 Equilibria in Oral Ascending Auctions

We now describe equilibria in a stylized version of an oral ascending auction. The oral auctions conducted by the Forest Service are "free-form": after each bid is placed (where each bid must specify bids for each species, not just the total bid), any bidder can choose to raise the bid. A bidder may place a bid, stay silent for a time, and then become active again.[9] If more than one bidder indicates a desire to bid, the auctioneer selects one. Importantly, a bidder is not permitted to raise her own bid at the end of the auction. If an auction ends, the high bidder is stuck with the skew they have chosen.

One concern with oral auction data is that announced bids may not be accurate indicators of beliefs, and in particular, that non-winning bids may be difficult to interpret. In the scale context, this problem might be acute since bidders have significant leeway to engage in strategic bidding behavior (such as skewing dramatically or onto the wrong species) early on to try to signal to or confuse the other bidders. Anecdotal evidence suggests these practices are not widespread, but nevertheless, we want to consider a more formal analysis of bidder behavior in oral auctions. In particular, we are interested in interpreting the skews of non-winning bidders, since these are observed in our data. We also wish to analyze the extent to which information is accumulated in the course of an oral auction, as many comparative statics results (for example, the prediction that the expected winning bid and winning skew will be larger in an oral auction than a first-price sealed bid auction) follow from this information accumulation.

Our oral auction model is similar to Milgrom and Weber's (1982, henceforth MW) "English" auction, which requires total bids to rise smoothly and does not permit bidders to drop out of the

[9]The Forest Service does have a rule stating that a bidder who has placed a bid earlier in the auction cannot *lower* his bid on any given species. Past bids by *other* bidders do not place any restriction on a given bidder's behavior, except for the standard requirement that total bids must go up.

bidding and re-enter.

(English scale auction) All bidders are active at zero. The auctioneer raises the total bid in small increments. At any bid increment, all bidders call out a skew (Δb) for the next increment. They then announce their activity for the next increment. No bidder who has dropped out can become active again. If only one bidder announces activity, that bidder is declared the winner at her last announced bid.

Observe that relative to MW, the scale auction has an additional ambiguity. In the MW English auction, a bidder's only decision is whether to remain active — and by remaining active, he reveals only that his signal is above some lower bound. In the scale auction, each bidder must also announce a bid allocation. A bidder could conceivably reveal all of his information in the first round of the auction. A simple argument, however, suggests that in equilibrium, the release of information must be gradual. Suppose that when the total bid was B, each bidder's strategy was to announce a distinct bid allocation depending on their signal. For example, they might choose the value $\Delta b_\chi(B, d_\chi)$ that maximizes their expected utility conditional on their signal and on winning at the next increment. Then each bidder's information could be immediately inferred by all opponents. From that point on, all bidders would have the same information and thus the same conditional expected value for the object. All bidders would bid until the expected utility from winning is zero, conditional on the signals of all of the bidders. This cannot be an equilibrium, since any individual bidder has an incentive to deviate by announcing a lower signal. Doing so would reduce the drop-out level of all opponents, and give the deviator positive expected profits. We conclude that there must be some "pooling" of signal types in equilibrium.

Given a history and a bid level B, we define the "marginal" private signal d_χ to be the signal that leaves a bidder indifferent between winning and losing at B, when the skew is chosen to maximize expected utility conditional on winning at B. We propose an equilibrium whereby, at each point in time, all active bidders mimic the optimal bid allocation announcement of a firm with the marginal signal. Further, bidders drop out at exactly the point where their signal becomes marginal, so that bidders with higher signals stay in the auction longer. Thus, our equilibrium has the same qualitative features as the MW equilibrium: (1) bidders with lower signals drop out first, and fully reveal their information when they drop out; (2) bidders remain in the auction until they are just indifferent between dropping out and winning the auction at the current bid; (3) the only information that can be inferred about active bidders is that their signals are at least as great as some marginal signal.

Formally, order the bidders in descending signal order. If k bidders have dropped out, let $P_k = \{p_1, ..., p_k\}$ be the vector of total bids at which the drop-outs occurred. Then, define the

function $B^*(D;k,P_k)$ as the solution to the following problem:

$$0 = \max_{\Delta b_\chi} E_{\delta_\chi}\left[u\left(Q_{ACT}\cdot\left((\Delta b-\Delta v)\delta+\frac{V-B}{Q_{EST}}\right)\right) \mid d_\chi^1 = ... = d_\chi^{J-k} = D, P_k, \chi\right] \quad (2)$$
$$\textit{subject to} \ : \ (B/Q_{EST}) + \Delta b_\chi(1-x_1) \geq r_1, \ (B/Q_{EST}) - \Delta b_\chi x_1 \geq r_2.$$

Given earlier drop-out prices P_k, the function $B^*(D;k,P_k)$ corresponds to the total bid level at which a bidder with signal $d_\chi = D$ can achieve a payoff of at most zero given that all other remaining bidders also have signal D.

Proposition 4 *$B^*(D;k,P_k)$ is strictly increasing in D, and $B^*(D;k,P_k) \geq V$.*

Defining $D^*(B;k,P_k)$ to be the inverse of B^*, we have now identified the "marginal" type given drop-out points P_k, and bid level B. Let $\Delta b^E(B;k,P_k)$ be the bid difference that delivers the solution $D^*(B;k,P_k)$. We can describe behavior in the auction as follows:

Proposition 5 *Assume that utility functions are CARA or IARA. (i) There exists a Perfect Bayesian equilibrium of the English scale sale auction with the following bidding strategies. Given that k bidders have dropped out and until another opponent drops out, each bidder stays in the auction until $B = B^*(d_\chi^j;k,P_k)$. For each $B < B^*(d_\chi^j;k,P_k)$, active bidders announce $\Delta b^E(B;k,P_k)$. (ii) The player with the highest signal wins the auction, and the $(\Delta b, B)$ that wins the auction solves*

$$0 = \max_{\Delta b_\chi} E_{\delta_\chi}\left[u\left(Q_{ACT}\cdot\left((\Delta b-\Delta v)\delta+\frac{V-B}{Q_{EST}}\right)\right) \mid d_\chi^1 = d_\chi^2, d_\chi^2, ..., d_\chi^J, \chi\right] \quad (3)$$
$$\textit{subject to}: \ (B/Q_{EST}) + \Delta b_\chi x_{\neg\chi} \geq r_\chi, \ (B/Q_{EST}) - \Delta b_\chi x_\chi \geq r_{\neg\chi}$$

Once again, the equilibrium has the property that bidders must skew to win — any winning total bid is greater than the maximum full-insurance bid with positive profits. It also has the feature that as the bid level rises, skewing becomes more aggressive, until only the bidder with the highest signal is willing to own the contract.

Though bidders can make a wide variety of deviations from the equilibrium strategies, the off-equilibrium-path beliefs required to support the equilibrium are weak. The main restriction is that following a larger than expected skew by a given bidder, marginal opponents, i.e. those who are about to drop out, must become slightly optimistic about that bidder's signal. The idea is that announcing a deviant bid allocation is payoff-relevant only if a bidder wins at that exact bid, that is, only if all opponents the marginal signal $D^*(B,P_k,k)$. If a deviation makes marginal bidders more optimistic, they will stay in the auction, and no gain can be realized. The assumption of increased optimism is reasonable since a bidder with a signal less than or equal to $D^*(B,P_k,k)$ would lose money if the auction ended following a deviation.

As mentioned above, the real-world Forest Service auction differs from our theoretical model in several ways. Bidders can drop out and re-enter the bidding and the bidding may rise in jumps. However, even in a more "free-form" model — where bidders do not necessarily have to indicate their activity at every point and where arbitrary jumps are possible — the behavior described above can be supported in a Perfect Bayesian Equilibrium given strong enough assumptions about off-equilibrium-path beliefs.

Consider an auction similar to the one above, except that after bidders indicate activity at a given round (for example, by raising their hands), one bidder is recognized and allowed to raise the total bid and call out an arbitrary skew. Bidders then indicate their activity for the next round, and the process repeats. Moreover, bidders may drop out (not announce activity at a given bid) and then re-enter later. In this auction, there will be an equilibrium in which bidders remain active until dropping out once and for all, raise the bid level only in the minimal increments, and skew according to $\Delta b^E(\cdot)$ as defined above. Essentially, two assumptions are sufficient to support this behavior as an equilibrium: following a "jump bid" (i.e. larger than some minimum increment), or an unexpected skew, opponents assign probability one to the deviant bidder having the highest possible signal. Moreover, if a bidder drops out and then re-enters, announcing activity at a later time, bidders similarly assign probability one to their having the highest possible signal. These off-path beliefs support the above strategies as an equilibrium, since there is no way for a deviator to make positive expected profits.

Although bidding behavior is unlikely to proceed in such a structured way in practice, the main qualitative feature of the equilibrium is somewhat more robust: bidders announce skews that are consistent with having observed the lowest possible signal that makes them profitable if they win.[10] This feature is natural because inducing more optimistic beliefs by the opponents simply leads them to bid more aggressively. It is particularly useful for interpreting the data recorded by the Forest Service. The FS simply reports the highest bid placed by a particular bidder. Since that bidder might have been willing to bid at higher prices than the last recorded price, this bid can serve only as a bound on the bidder's total value. However, if the bidder is playing according to the equilibrium described above, the bid *does* have an exact interpretation. The losing bid allocation can be interpreted as the optimal choice by a bidder whose signal is marginal for the level of the total bid. Although we will not pursue it here, this fact could be used to recover parametric estimates of the underlying risk aversion and signal distribution in a structural model.

[10]Equilibria with this feature also could be constructed with different rules about activity announcement, as in Harstad and Rothkopf's (1997) "alternating recognition" model, where only two bidders at a time announce activity.

3.3 Predictions of the Theory

We now summarize a number of theoretical predictions that we will attempt to check against the data. Since the private signals of the bidders are not observable, we frame our predictions in terms of the relationship between the observed bids and the Forest Service mis-estimates (the difference between the estimated and actual species proportions). From the perspective of the econometrician, both the mis-estimate, $\delta = x_1 - \rho_1$, and the observed bids, B and $\Delta b = b_1 - b_2$, are random variables. The theoretical model predicts a systematic relationship, summarized in Proposition 6.

Proposition 6 *Assume that utility functions are CARA or IARA. In the oral and sealed bid auction, bidder j's equilibrium choice of Δb^j is affiliated with δ and the same conclusion holds conditional on any fixed total bid B^j.*

We now briefly summarize a number of comparative statics implied by the model. To state some of the predictions, it is useful to order the species so that the Forest Service mis-estimate is positive. Let ϕ be the species that is over-estimated, so that $\delta_\phi = x_\phi - \rho_\phi \geq 0$ and $\Delta b_\phi = b_\phi - b_{\neg\phi}$. Observe that Proposition 6 holds no matter how we choose to order the species; $\delta \cdot \Delta b^j$ is positive exactly when the bid and the skew are in the same direction.

1. The amount bid per unit of volume is expected to be greater than the amount paid per unit of volume, i.e. $E[B^j/Q_{EST} - P^j/Q_{ACT}] = E[\Delta b^j \cdot \delta] > 0$.

2. On average, bidders skew in the "right" direction: $\Pr\{\Delta b^j_\phi - \Delta v_\phi > 0\} > \frac{1}{2}$; and further, $\Pr\{\Delta b^j_\phi - \Delta v_\phi > 0 | \delta_\phi\}$ is increasing in δ_ϕ.

3. The expected skew will be nondecreasing in the mis-estimate, i.e. $E[\Delta b^j - \Delta v | \delta]$ is increasing in δ; and similarly for a fixed total bid B^j, $E[\Delta b^j - \Delta v | B^j, \delta]$ is increasing in δ.

4. The skews of the higher ranked bidders should be larger, both unconditionally and for any given bid level B, i.e. $E[\Delta b^j_\phi - \Delta v_\phi]$, $E[\Delta b^j_\phi - \Delta v_\phi | B^j]$ are decreasing in j.

5. Conditional on the per-unit bid, the amount paid per unit of volume, $E[P^j/Q_{ACT} | B^j/Q_{EST}, \delta_\phi]$, is decreasing in the over-estimate δ_ϕ.

6. Under CARA, the second-highest bid should be more skewed in an oral than in a sealed bid auction, i.e. $E[\Delta b^1_\phi - \Delta v_\phi | \delta_\phi, ORAL] - E[\Delta b^1_\phi - \Delta v_\phi | \delta_\phi, SEALED]$ is increasing in δ_ϕ.

4 Data on Forest Service Auctions

4.1 The Data

Our data set includes a subset of the bidding and cutting data from Forest Service timber sales described in Section 2. To our knowledge, we are the first to combine bidding and cutting data to systematically analyze bidding behavior in timber auctions.[11] We restrict attention to "two species" sales, where two species, but no more than two, each comprise at least 25% of the volume on the tract. We have found the most transparent way to think about skewing is in the context of two-species sales because bidders can skew their bids only along a single dimension.[12] In much of our analysis, we restrict attention to oral auctions in Region 6 (Oregon and Washington) that required only a small amount of road construction.[13] In Section 5.4, we consider sealed bid sales in Region 5 (California).

In the bidding data, we observe all information that the Forest Service makes public after its cruise: its estimated volumes of timber, the reserve price, and also its estimates of end-product selling values and projected processing costs. The bidding data also includes the identities of all bidders and their bids. In the oral auctions, the bid we observe for bidder j is that bidder's last announced bid.[14] The cutting data allows us to observe the total volume and species proportions actually removed from the tract (these are the same records used to bill the winning bidder).

In our empirical analysis of skewing and revenue, we include a number of control variables, referred to as $\mathbf{X}$. These include the volume of the tract and the average reserve price, whether or not the sale was a small-business set-aside sale, and controls for other sale characteristics. These include the number of months in the contract per unit of volume; the density of timber (where a higher density may indicate "old-growth" and thus more variable volumes); the volume of per-acre material, which is essentially scrap; the estimated logging costs for the tract; and the estimated amount of road construction required. A variety of additional sale characteristics are also available in the data; the results we report are robust to our choice of controls. In our analyses of skewing (but not revenue), we control for the number of bidders.[15] Last, we include dummy variables for

[11] Although see Cummins (1995), who uses the cutting data to analyze how the timing of timber harvesting varies with market prices through the life of the timber contract.

[12] When analyzing skews, we simply ignore any bids and/or mis-estimates on the remaining species. In contrast, when we study revenue, we consider bids on all species.

[13] In Forest Service auctions, bidders are "reimbursed" for road-building using a system of credits that can be redeemed for timber. Restricting attention to sales where road construction is minimal allows us to more directly interpret the prices paid by the bidders.

[14] As noted above, a bidder's last observed bid might bear only a distant relationship to the highest bid the bidder is willing to place, but our theoretical model gives some guidance in interpreting the data: even if the bidder is willing to place much higher bids, the observed skew is optimal for some "marginal" signal.

[15] Our results about the number of bidders should be treated with some caution, since unobserved features of the

the time period, for several large forests, and for several common species combinations.

Appendix B provides further details about our data sources and sample selection criteria. Table I contains summary statistics.

4.2 Preliminary Observations

On average, the revenue the Forest Service collects from a given sale is around 9% (in our main sample, \$40,692) less than the winning bid. To see how the gap between winning bid and revenue arises, we can decompose it as follows:

$$B - P = (B/Q_{EST})(Q_{EST} - Q_{ACT}) + (\Delta b \cdot \delta) \cdot Q_{ACT},$$

where the first term is the portion of the bid-revenue gap due to the volume shortfall (on average, the volume cut is 5.8% less than the volume estimated), and the second term is due to the bidder systematically paying a different amount than he bid for a "representative tree" from the tract. In our sample, the first term accounts for \$30,110 of the \$40,692 bid-revenue gap, while the second terms averages out to \$10,582. Thus for the average thousand board feet of timber removed from a tract, the auction winner pays about \$4 less than it bid (the average winning bid is \$143).

A second feature of the data is the significant variance in the Forest Service estimates and in the amount of skewing. Chart I shows the distribution of Forest Service mis-estimates, demonstrating the potential return to information. The Forest Service estimates of the species proportions differs from the proportions removed by more than 5% in nearly half the sales (45%) and by more than 10% in about one of six sales (17%). Chart II shows how the winner distributed his or her overbid (the difference between the total bid and the reserve price) among the two species. As established in the theoretical analysis, a risk-neutral bidder with information would place his or her entire overbid on one species or the other, but the chart shows significant dispersion and many sales where the winning overbid is distributed across both species.

Finally, we note one aspect of the data that is, strictly speaking, inconsistent with our model. While our model assumes the existence of enough public information so that in equilibrium all bidders skew in the same direction, this is clearly not the case in practice. Chart III shows a scatter-plot of the skew of the winning and second-place bids (III.A), and of the winning and

tract may affect both participation and skewing behavior. However, our feeling is that for skewing regressions, this endogeneity problem is less significant than in standard analyses of auctions, where the main dependent variable is the magnitude of the bid. Here, the unexplained portion of a bidder's skew should correspond to private information obtained during the cruise. If one thinks of the cruise as tantamount to an entry decision, it is plausible that unobserved factors leading to entry will be unrelated to information obtained after the decision to cruise is made. In any event, our results are robust to both the omission of the number of bidders as an explanatory variable, as well as to instrumenting for the number of bidders using some of the standard approaches from the existing literature (such as the forest and district of the tract and related measures, as in Hansen (1985) and Haile (1998)).

third-place bids (III.B). Both scatter-plots exhibit a clear positive dependence but also significant variation. In some auctions, bidders skew in *entirely opposite* directions: one bidder has all of the overbid on species 1, the other on species 2.

4.3 Bid Skewing: Some Examples

To give some sense of the pattern of bid allocation in oral auctions, we briefly discuss a few "case studies" of bidding in oral auctions. In our theoretical model, skews increase monotonically with oral auction bids. While this is the case in many sales in the data, the it is by no means the rule. Moreover, in many auctions, the reserve price plays an important role — at least one of the bidders places his entire overbid on one species, so that the reserve prices are binding. In fact, there are relatively few examples of auctions where *every* observed bid is an intermediate skew, and the skews are ordered exactly as predicted by theory. The following is one such case:

Example: Monotonic Bidding Behavior. $\delta = .019$

Bidder	B/Q_{EST}	Sp. 1 skew: $\Delta b - \Delta r$	% overbid on sp. 1	$(\Delta b - \Delta r) \cdot \delta_1$
1	347.8	339.39	.94	6.45
2	346.5	334.39	.94	6.35
3	285.6	50.39	.56	.96
4	277.5	33.39	.52	.63
5	250.72	.39	.48	.007

On occasion, skewing can have very large consequences. Consider the following sale where the bidders skewed heavily in one direction at the start, then switched later in the auction.

Example: Switching Skews. $\delta = .227$

Bidder	B/Q_{EST}	Sp. 1 skew: $\Delta b - \Delta r$	% overbid on sp. 1	$(\Delta b - \Delta r) \cdot \delta_1$
1	234.3	523.96	1	118.9
2	233.9	522.96	1	118.7
3	207.6	−396.62	0	−90
4	193.8	−353.66	0	−80

In this example, earlier bids are skewed into species 2, which was in fact under-estimated. As it turned out, the average payment was less than half of the winning bid; if the third or fourth bidder had been awarded the contract, they would have paid significantly more than their average bid. The Forest Service would have received more revenue from awarding the tract to one of the lower bidders.

There are a small number of cases where bidders seemed to place a "surprise" skew at the end of an auction. Boise Cascade did this in two instances. In each instance, several lower-ranked bidders had placed large skews in one direction, and Boise Cascade won the auction with a very small increase in the average bid, but with a switch in the skew of several hundred dollars. In one case, the skew turned out to be in the right direction; in the other, it was wrong. It appears that information affects allocation of the tract, since the winner pays a very different price than the next-lower bidder would have for the same object, and the winner's behavior reveals that the winner anticipates this.

5 Evidence on Skewing

We now turn to testing whether the predictions of the theory are consistent with the Forest Service data. Before presenting our empirical analysis, we discuss two choices we made that merit discussion.

First, the theoretical model suggests thinking about the skewing relative to a "full-insurance" allocation; that is, the variable of interest is $\Delta b - \Delta v$. However, the data contain no exact analogue to the "values" in our model. We use the difference in reserve prices as a proxy for the difference in the values of the species, i.e. we assume that $\Delta v = \Delta r$. In a linear regression of Δb on Δr, this amounts to a restriction on the coefficient of Δr. Such a restriction seems reasonable — in a subsample of 97 auctions where mis-estimates are small and the reserve prices do not bind, a regression of Δb on Δr leads to a coefficient of 1.055 on Δr (with a standard deviation of .171 and an R^2 of 0.28).

A second point concerns the way in which we order the species. If we were interested only in the unconditional relationship between Δb and δ, the order of the species would be irrelevant (because $(\Delta b_1, \delta_1) = -(\Delta b_2, \delta_2)$). Yet we would like to control for the possibility that some observable features of the tract might explain the fact that bidders skewed in the right direction. This suggests studying the relationship between the absolute magnitude of the mis-estimate ($\delta_\phi > 0$) and the skew in the "right" direction $\Delta b_\phi - \Delta r_\phi$. We follow this approach in order to test the model's predictions. In Section 6, we consider an alternative approach: namely, we assume that the model is correct, infer χ (the species that the winning bidder believes is most likely to be over-estimated), and then directly estimate bidding functions $\Delta b_\chi(\cdot), B(\cdot)$.

5.1 Winning Bids Reflect ex ante Information

We begin with the primary question of whether winning bids in Forest Service auctions reflect *ex ante* information about Forest Service mis-estimates. The following cross-tabulation suggests that

winning bids tend to be skewed in the right direction, especially when the Forest Service makes a significant mistake in estimating the species proportions.

Mis-estimates and Skews in Region 6 Oral Auctions

	Incorrect Skew ($\Delta b_\phi - \Delta r_\phi < -20$)	Small Skew ($-20 \leq \Delta b_\phi - \Delta r_\phi \leq 20$)	Correct Skew ($\Delta b_\phi - \Delta r_\phi > 20$)
Small mis-estimate ($\delta_\phi < .025$)	83	44	90
Larger mis-estimate ($\delta_\phi > .025$)	139	104	239

Overall, the winning bid is skewed correctly ($\Delta b_\phi - \Delta r_\phi \geq 0$) in 58% of the sales. Table 2 reports a probit regression, confirming the theoretical prediction that, conditional on the control variables ($\mathbf{X}$), larger mis-estimates correspond to a higher probability that the winning bid is skewed in the right direction. An increase in the magnitude of the mis-estimate of .01 is associated with roughly a 1-2% higher chance of the winning bid being skewed correctly. Column (2) establishes that the relationship is concave: the initial effect of an increase of .01 in the mis-estimate is a 2-4% increase in the probability of a correct skew, but the slope diminishes in δ_ϕ. Of the control variables, the most interesting result is that the number of bidders increases the probability of skewing in the right direction. This finding supports the hypothesis that information is revealed in the auction.[16]

To obtain a more rigorous test of whether bidders in fact have ex ante signals about the ex post value beyond what can be publicly observed, we would like to account for the competing hypothesis that Forest Service estimates are systematically biased, and bidders have realized this and just skew systematically in response. To do this, we ask whether *unanticipated* deviations in δ help to explain the winning bid allocation, controlling for exogenous sale characteristics ($\mathbf{X}$) that might "predict" the mis-estimate. However, theoretically we have no reason to expect a linear relationship between $\mathbf{X}$ and the skew; $\mathbf{X}$ might affect the bidder's risk aversion, or her beliefs, in a variety of ways. Since empirically, $\mathbf{X}$ is correlated with δ_ϕ, it is important to isolate the effect of the mis-estimate from a potential nonlinear relationship between $\mathbf{X}$ and the distribution of δ_ϕ. To accomplish this, we consider the following model:

$$\Delta b_\phi - \Delta r_\phi = h(\mathbf{X}) + \delta_\phi \gamma + \varepsilon, \tag{4}$$

where $\mathbf{X}$ contains sale characteristics as described above. If the bidders do not have ex ante information about δ_ϕ, we should find $\gamma = 0$.

[16]As dicussed in footnote 15, it seems plausible that the number of bidders is exogenous in the skewing regression, as bidders must cruise the tract to learn about mis-estimates. Our results are robust to excluding this control.

We consider a semi-parametric approach to estimation. Dividing the control variables $\mathbf{X}$ (as listed in Table 2) into continuous regressors $\mathbf{Z}$ and dummy variables $\mathbf{W}$, we allow $h(\mathbf{X}) = \hat{h}(\mathbf{Z}) + \mathbf{W}\boldsymbol{\beta}$, where $\hat{h}$ is an arbitrary continuous function of two indices $(\mathbf{Z}_1\boldsymbol{\alpha}_1, \mathbf{Z}_2\boldsymbol{\alpha}_2)$. We begin by estimating

$$\Delta b_\phi - \Delta r_\phi = \hat{h}_\Delta(\mathbf{Z}) + \varepsilon_\Delta,\ W = \hat{h}_W(\mathbf{Z}) + \varepsilon_W \text{ and } \delta_\phi = \hat{h}_\delta(\mathbf{Z}) + \varepsilon_\delta, \tag{5}$$

letting $\hat{h}_\Delta$, $\hat{h}_W$ and $\hat{h}_\delta$ take a double-index form.[17] While it should be noted that our double-index model is still somewhat restrictive, our results are robust across a wide range of specifications of the two indices $\mathbf{Z}_1$ and $\mathbf{Z}_2$. In the specifications we report, we let $\mathbf{Z}_1$ be the average reserve price, while $\mathbf{Z}_2$ contains the remaining continuous control variables. Denoting the residuals from these semi-parametric regressions as $e_\Delta, \mathbf{e}_W$ and ε_δ, we then use ordinary least squares[18] to estimate

$$e_\Delta = \mathbf{e}_W\boldsymbol{\beta} + e_\delta\gamma + \varepsilon. \tag{6}$$

Our estimate of the coefficient γ is 406.8 with a standard error of 114.8. That is, an increase in the mis-estimate of .01 is associated with a \$4 increase in the skew (in the right direction). We can reject the null hypothesis that $\gamma = 0$ at the 1% level. These findings can be compared with a simple linear specification, assuming $h(\mathbf{X}) = \mathbf{X}\boldsymbol{\beta}$. As shown in Table III (2), this yields an OLS estimate of γ of 383.8 with a standard error of 140.1. Table III (3) includes a control for the magnitude of the bid; this captures the idea that part of the effect of the mis-estimate is to encourage more aggressive bidding. Indeed, we find a positive, significant effect of the value bid on the skew, alongside a somewhat smaller point estimate of γ (319.6 with a standard error of 121.9). However, we are cautious about interpreting these coefficients as structural parameters of the bidding function, due to the fact that we have ignored the issue of the reserve prices. The per-species reserve prices bind in about 42% of our observations; this implies that bidders were not able to skew as much as they would have liked, without further raising their overbid. Further, the magnitude of the overbid, an endogenous variable that depends on δ_ϕ, determines whether or not the reserve prices bind. Thus, we interpret the coefficients in Table III as reduced-form correlations, deferring a more formal analysis of the reserve prices to Section 6.

Finally, all of these specifications restrict the relationship between $(\Delta b_\phi - \Delta r_\phi)$ and δ_ϕ to be linear. While establishing a linear relationship is sufficient for testing our hypotheses about bidder

[17] To estimate, e.g., $\Delta b_\phi - \Delta r_\phi = \hat{h}_\Delta(\mathbf{Z}) + \varepsilon_\Delta$, we used the average derivative method of Powell, Stock and Stoker (1989) to obtain consistent (up to scale) estimates of $\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2$, where $\hat{h}_\Delta(\mathbf{Z}) = \hat{h}_\Delta(\mathbf{Z}_1\boldsymbol{\alpha}_1, \mathbf{Z}_2\boldsymbol{\alpha}_2)$. We then then used a kernel estimator (with a normal density) to estimate the function $\hat{h}_\Delta$, estimating $\Delta b_\phi - \Delta r_\phi = \hat{h}_\Delta(\mathbf{Z}_1\hat{\boldsymbol{\alpha}}_1, \mathbf{Z}_2\hat{\boldsymbol{\alpha}}_2) + \varepsilon_\Delta$. We used a routine described in Stoker (1989) to choose the bandwidth (the values were .41 for the first index and 1.1 for the second), and we used a trimming rule of 5%. Tom Stoker generously provided portions of code for the estimation.

[18] As Robinson (1988) points out, OLS estimation of e_Δ on $\mathbf{e}_W$ and e_δ yields consistent estimates of $\boldsymbol{\beta}$ and γ and the OLS standard errors are correct. See also Stoker (1989) for a clear discussion of the model we consider.

information, it is interesting to inquire more generally into the relationship between skews and mis-estimates. To do this, we estimated the model $\Delta b_\phi - \Delta r_\phi = h(\mathbf{Z}\alpha, \delta_\phi) + \mathbf{W}\beta + \varepsilon$ (where $\mathbf{Z} = (\mathbf{Z}_1, \mathbf{Z}_2)$ and $\mathbf{W}$ are defined as above), using the density-weighted average derivative method of Powell, Stock and Stoker (1989) as described above. The main finding, robust across a variety of specifications of the index $\mathbf{Z}$, is that the average derivative with respect to δ_ϕ is positive and statistically significant at a 5% level of confidence. To see the shape of the functional relationship, Chart IV plots the resulting estimate of the relationship between the skew and the mis-estimate ($h(\mathbf{Z}\alpha, \delta_\phi)$, evaluated at the mean of $\mathbf{Z}$). The chart illustrates the positive slope and a generally convex shape over the region of δ_ϕ with most of the density weight.

5.2 Bidders have Different (Private) Information

A key aspect of our skewing model, and indeed of any "common-value" auction, is that allocation is driven by private information rather than preferences.[19] In our model, bidders who are more optimistic about the gap between bids and payments skew more aggressively and submit higher bids. Thus, we would like to know in the data whether higher-ranked bidders were more aggressive in their skews. We also seek to test whether higher-ranked bids are more likely to be skewed in the right direction: because information is revealed during the course of an oral auction, the j^{th} highest bid should incorporate strictly more information than the $(j+1)^{st}$.[20] Finally, our model of information revelation implies that the top two bidders should exhibit very similar skewing behavior. The top bidder should stay in the auction for only one bid increment past the second-highest bider, while lower-ranked bidders will in expectation drop out at significantly lower bid levels with significantly less aggressive skews.

To investigate whether higher-ranked bidders allocate their bids more accurately, we look at the following model:

$$\text{Dummy(Skew in right direction)}_{jt} = \sum_{j=1}^{4} \alpha_j \cdot \text{Dummy(Rank } j) + \mathbf{X}_t\boldsymbol{\beta} + u_t + \varepsilon_{jt}. \tag{7}$$

We estimated this as a fixed-effects linear probability model. Using a fixed effect for each auction eliminates the auction-specific sale characteristics and the disturbance u_t, allowing us to isolate the effect of the rank. The results are reported in Table IV. As expected, we find that the winning and

[19] Industry sources reported complaints by bidders that mis-allocation results when their opponents have "bad" information (i.e. were "over-optimistic" about the skew).

[20] As we noted in Section 4.2, our theoretical model is not consistent with the data on this point, since in equilibrium, it suggests that all bidders will skew in the same direction (which is not the case in practice). However, the result that bid j in an oral auction will be based on more information than bid $j+1$ should be robust across a wider range of models.

second-highest bid are significantly more likely to be skewed in the right direction than the third and fourth-highest bids.

The second question is whether the skews of higher-ranked bidders are more aggressive. Here, we restricted attention to sales where none of the top four bids is censored by the reserve price.[21] We estimated

$$(\Delta b_\phi - \Delta r_\phi)_{jt} = \sum_{j=1}^{4} \alpha_j \cdot \text{Dummy(Rank } j) + \mathrm{X}_t\beta + u_t + \varepsilon_{jt}, \tag{8}$$

again using a fixed effects specification. As is shown in Table IV (2), the winning and second-highest bids were similarly skewed, while the third and fourth-highest bids were significantly less skewed (by roughly \$20–30).[22] The third column of Table IV reports a final specification where the dependent variable is an alternative measure of aggressiveness: the absolute magnitude of the bidder's skew. This specification separates out the fact that higher-ranked bidders skew more accurately and concentrate solely on the size of the skew. We again find that the top two bids are very close, while the third and fourth bids are less skewed. In this case, there is a substantial difference between the magnitude of the third and fourth-highest bids; both are significantly less skewed than the top bids.

Our findings are consistent with the hypothesis of differential information among the bidders. Suppose, alternatively, that bidders have identical beliefs about the true volumes on the tract and choose optimal decision-theoretic skews. In this case, observed differences in bids and skews might be driven by differences in costs, private values for the species, or risk aversion (either intrinsic or due to different bid levels and values for the tract). For instance, if bidders have decreasing absolute risk aversion, then bidders with a higher overall value for the tract would be willing to skew more aggressively.[23] Such a model has higher-ranked bids being more skewed, though all bids skewed in the same direction. Idiosyncratic variation in Δv also could generate variation in skewing, simply through differences between the "full-insurance" allocation.

These private value explanations have number of shortcomings. First, there is no reason to think higher-ranked bids should be more accurate. Second, there are numerous cases in the data where we observe a given bidder bidding on some combination of two species (e.g. hemlock and fir) in multiple sales and skewing in different directions in different sales. Third, reconciling the

[21] Although this restriction leads to a non-representative sample, it is still possible to test predictions about differences in behavior across ranks within an auction. We expect that the sample of uncensored auctions displays less pronounced skewing behavior than the full sample.

[22] One concern is that we have relatively few sales in which none of the top four bidders were constrained by the reserve prices. We performed a robustness check by enlarging the sample to include all sales where the top three bidders were unconstrained. We obtained similar results — no significant distinction between the top two bidders; the third-highest bid tends to be closer to full-insurance.

[23] We are grateful to Phil Haile for pointing out this potential explanation for monotonicity of the skews.

observed data with differences in private values or risk parameters seems to require implausible differences across bidders. The fact that bidders can resell timber and subcontract logging suggests some compression in values for the different species. Yet, among the subset of 72 auctions in which the top four bids are not close to the reserve price, the average difference between the largest and smallest skew was substantial — \$46 (with a standard deviation of \$86 and maximum of \$393). To place this in perspective, the average winning bid was \$167, while the average magnitude of the winning skew was \$87. Finally, while our information-based model of bid allocation is consistent with the similar skews of the top two bidders relative to the others, it is hard to see how this could be explained in a model without some aspect of differential information.

We draw three conclusions regarding the variation in skews across bidders. First, dispersion in skews can be large within a given sale. Second, it appears that bids later in the auction are "more informed." This might reflect either more accurate prior estimates or the fact that information is acquired during the auction. Third, given that winning bidders are effectively paying a lower percentage of their bids than lower-ranked bidders, the logical consequence of our findings is that information should have an effect on allocation.

5.3 Revenue Effects

One simple consequence of the scale sale format is that losing bidders might have actually generated more revenue than the winners, given the actual volumes cut. In our sample, a losing bidder would have generated more revenue in 17% of the sales; the revenue "loss" from this misallocation is \$5.39 million, about 1.7% of the total revenue. While such discrepancies have attracted the attention of policy-makers (see GAO report RCED-83-37), such magnitudes should be interpreted with caution: the losing bids as well as the winning bids were placed taking into account the benefits of skewing, so clearly, any change in the allocation rule would also change the equilibrium bidding strategies.

Given our model, it is interesting to ask how revenue changes in response to an arbitrary Forest Service error, that is when δ_ϕ increases. With two species, the revenue collected by the Forest Service per unit of volume is equal to $B/Q_{EST} - \Delta b_\phi \cdot \delta_\phi$, so the change in revenue is

$$\frac{\partial}{\partial \delta_\phi}\text{Revenue} = \frac{\partial}{\partial \delta_\phi} B/Q_{EST} - \frac{\partial}{\partial \delta_\phi}\Delta b_\phi \cdot \delta_\phi - \Delta b_\phi. \tag{9}$$

An increase in δ_ϕ affects both the magnitude of the bid and the gap between bid and payment, $\Delta b_\phi \cdot \delta_\phi$. Consider first the latter effect. We have already established that $\frac{\partial}{\partial \delta_\phi}\Delta b_\phi > 0$, a force in favor of a reduction in revenue. However, the sign of Δb_ϕ is ambiguous. If Δr_ϕ is greater than 0, then on average we expect $\Delta b_\phi > 0$ as well. But, if $\Delta r_\phi < 0$, we cannot sign Δb_ϕ. In our sample, when $\Delta r_\phi \geq 0$, the mean of $\Delta b_\phi = \$99$; when $\Delta r_\phi < 0$, the mean of $\Delta b_\phi = -\$46$. To capture the net effect of the mis-estimate on the gap between the bid and the payment, Table V (1) reports

the results from the following specification:[24]

$$B/Q_{EST} - P/Q_{ACT} = \delta_\phi \mathbf{1}_{\{\Delta r_\phi > 0\}}\gamma_1 + \delta_\phi \mathbf{1}_{\{\Delta r_\phi < 0\}}\gamma_2 + \mathbf{X}\beta + \varepsilon. \tag{10}$$

Overall, the gap between the bid and the payment is increasing in δ_ϕ irrespective of the sign of Δr_ϕ, but derivative is larger when $\Delta r_\phi \geq 0$ than when $\Delta r_\phi < 0$ (\$110 with a standard error of 34, versus \$82 with a standard error of 24).

Now consider $\frac{\partial}{\partial \delta_\phi} B/Q_{EST}$. Our theoretical analysis established that the level of the bid increases when δ_χ increases. However, it is not necessarily true that $\phi = \chi$. As analyzed above, $\Pr(\phi = \chi|\delta_\phi)$ is nondecreasing in δ_ϕ; but, for those cases where $\phi \neq \chi$, then δ_ϕ is affiliated with $-d_\chi$. A higher δ_ϕ leads such bidders to be less optimistic about the magnitude of the mis-estimate, and to bid less. Overall, if $\Pr(\phi = \chi|\delta_\phi)$ is high enough, the expected value of the winning bid should be increasing in the actual mis-estimate δ_ϕ; but, in general the relationship may be ambiguous.[25] Table V (2) reports a regression of B/Q_{EST} on sale characteristics, following the specification from (10). We see that when $\Delta r_\phi < 0$, higher mis-estimates lead to higher bids, but not when $\Delta r_\phi \geq 0$.

Finally, to estimate the net effect of a Forest Service mis-estimate on revenue, in Table V (3) we report the relationship between revenue (P/Q_{ACT}) and the mis-estimate, following the same specifications as above. An increase in the mis-estimate, conditional on $\Delta r_\phi \geq 0$, is associated with a statistically significant reduction in revenue. The effect is relatively small in magnitude (a mis-estimate of .05 is associated with roughly a \$12,500 reduction in revenue). Conditional on the lower-valued species being over-estimated, a larger mis-estimate is associated with an (even smaller in magnitude) *increase* in revenue.

Summarizing, mis-estimates appear to have a negative effect on the revenue received by the Forest Service. However, we mention several caveats. The first is functional form; the specifications include only linear functions of $\mathbf{X}$ and δ_ϕ. While the result about revenue loss is only reinforced when we allow for richer functional forms for $\mathbf{X}$,[26] the functional relationship between B/Q_{EST} and δ_ϕ appears to be non-monotonic, and our estimates are somewhat sensitive to specification. Second, as discussed above, we have not accounted for the fact that the reserve prices constrain skewing behavior in 42% of the sales. We return to this issue in Section 6.

[24] As the sales in our sample have more than two species, $B/Q_{EST} - P/Q_{ACT} \neq \Delta b_\phi \cdot \delta_\phi$, although this is a close approximation. The empirical results report the gap between the bid and payment including all species.

[25] Even if $\frac{\partial}{\partial \delta_\phi} B/Q_{EST} \geq 0$, the magnitude of $\frac{\partial}{\partial \delta_\phi} B/Q_{EST}$ depends on a number of factors, including the risk aversion of bidders and the variance of δ.

[26] We also estimated the models of Table V (2) and (3) using the semi-parametric methods from Section 5.2, following the two-step procedure described in equations (5) and (6). In the bidding equation, our estimates of γ_1 and γ_2 are 110 with a standard error of 43, and -16 with a standard error of 39, respectively. In the revenue equation, our estimates of γ_1 and γ_2 are 49 with a standard error of 42, and -121 with a standard error of 37, respectively. Thus, relative to the linear model, these results suggest greater revenue loss.

5.4 Sealed Bid Auctions

We now report some evidence on sealed bid auctions. We are especially interested in sealed bid auctions because the interpretation of skewing behavior is unambiguous – each firm's bid is only relevant in the event that the firm wins, and so it is a dominant strategy for each bidder to choose the skew that maximizes expected utility conditional on winning and the bidder's private signal. Using a small (63 sales) sample of "two-species" sales from Region 5, we reprise the main tests conducted on the oral auction sample. We chose Region 5 because sealed-bid auctions were more prevalent there.

The first column of Table VI reports sealed bid estimates of the probit model from Section 5.1, where the dependent variable is the probability that the winner skewed correctly. Overall, the winner skewed in the right direction in 65% of the sealed bid sales, and was more likely to skew in the right direction when the mis-estimate was large. The size of the winner's skew $(\Delta b_\phi - \Delta r_\phi)$ is increasing in the mis-estimate δ_ϕ (Table VI (2)). An increase in the mis-estimate of .01 is associated with an additional \$3.80 skew in the right direction (our point estimate in the oral auction sample was \$3.83). Thus it appears that in sealed bid auctions, as well as in oral sales, the winner's bid does incorporate information about the species proportions beyond what is known from the Forest Service estimates.

The last three columns of Table VI report sealed bid estimates of the fixed-effects panel specifications, analogous to Section 5.2. We restricted the sample first to sales where there were at least three bids (column (3)) and then (columns (4)-(5)) to a still smaller sample where the top three bids are unconstrained by the reserve prices (that is, where the overbid was large enough relative to the skew that the reserve prices do not bind). Across ranks, we did not find significant differences in the accuracy of the skews, nor in the magnitude of the skews in the right direction $(\Delta b_\phi - \Delta r_\phi)$. We did, however, find that the winner's skew is significantly larger in absolute magnitude than the skews of the lower-ranked bidders. In fact, the magnitude of the skew is ordered by rank. These findings are fairly consistent with the logic of information-based skewing, though not perfectly so. Since there is no learning in a sealed bid auction, it is not clear that the winner should have more accurate information than other bidders. However, according to theory, the winner is more optimistic, and hence skews more aggressively, leading to larger absolute skews (which we found). Because bidders are likely to skew in the right direction, this should presumably also lead to larger skews in the right direction (which we did not observe).[27]

[27] One theoretical prediction that we were unable to test is that the skews of the second-highest bids should be larger in an oral sale. Our samples of Region 6 oral sales and Region 5 sealed bid sales are sufficiently different (Region 5 sales generally having younger, less-valuable, second-growth trees, and fewer bidders) that we did not feel confident making direct comparisons. Moreover, the choice of auction format is conditioned on sale characteristics and is endogenous. In exploratory work using a mixed sample of sales from Region 1 (Idaho and Montana), we did

5.5 Moral Hazard in Cutting: An Alternative Hypothesis

An alternative story, that can potentially explain many of the empirical regularities described above, is that bidders do not have private information *ex ante*, but instead are able to destroy some of the timber on the tract rather than pay for it. This would lead to a moral hazard problem, where sale winners would have incentives to destroy low-quality timber rather than harvest it.[28] Anecdotal accounts from the industry suggest that loggers may have a limited ability to manipulate the ex post measurement process. This would help to explain the 6% difference between the estimated and observed volumes.[29] Moral hazard could also explain observed skewing, since higher bids on a given species would imply a larger motivation to destroy low-quality logs of that species[30] — thus generating a correlation between skews and differences in estimated and cut species proportions. Of course, the presence of moral hazard is not inconsistent with our story of ex ante information and might *reinforce* incentives to acquire information — bidders may want to examine a tract to see whether it will be easy or difficult to destroy low-quality timber of a given species. We can place this in the framework of our model by thinking of easy-to-destroy low-quality timber as being simply not present. In general, the more skewed the bidding, the higher the incentives for ex post destruction of one of the species;[31] and, conversely, the easier it is to destroy low-quality trees of a particular species ex post the more incentive to skew.

Several pieces of evidence weight against an explanation of the data based solely on moral hazard. With pure moral hazard, the *total volume* of timber destroyed should be strongly correlated with the "observed mis-estimate." But in the data, the correlation between the proportions estimated and cut and the total volume estimated and cut is very low and insignificant: 0.053 (with a t-statistic of 0.32). This finding is confirmed when we control for observed sale characteristics.[32] Moreover, the distribution of skews within sales is hard to rationalize with a moral hazard story. We found in Section 5.2 that the second-highest bidder was just as likely to skew in the right direction as the winner, while the lower-ranked bidders were significantly less likely to skew in the right

find some difference between skewing behavior in oral and sealed sales, but perhaps due to the very small sample the results were sensitive to specification.

[28] A similar story has loggers colluding with the monitors responsible for measuring the timber as it leaves the tract.

[29] Leffler and Rucker (1988) analyze the effects of various policies on the harvest rates on Forest Service contracts, showing that a higher percentage of the estimated timber is extracted when contracts are shorter in duration.

[30] Only logs that meet a certain threshold are counted at all; broken limbs and other damaged material is charged at a different rate. Thus, a bidder could avoid "stealing" timber simply by breaking a log into pieces and paying a low price for it.

[31] Interestingly, one Forest Service official told us that cutting was monitored intensely when bids were especially skewed.

[32] We also looked at how the gap between estimated and cut total volume and species proportions varied over time. Because timber prices were quite volatile, one would expect the gap in total volume to vary a lot if moral hazard were a primary concern. On average, it remained relatively stable over time.

direction. This is sensible in the context of an ex ante information story, but if moral hazard were the primary force, the winning bidder's cutting behavior should respond to her own skew, while the second-highest bidder's skew should be less closely related to the actual difference in proportions.

5.6 Bidder Heterogeneity

Our theoretical model assumes that bidders are homogeneous. However, in our background discussion on the timber program, we raised the issue that bidders can be a very heterogeneous group, with giant conglomerates bidding against small independent logging operations. Because it seems plausible that there could be systematic differences between different type of bidders in terms of their skewing behavior, we briefly note our attempts to uncover observed differences between bidders that might be major (un-modeled) determinants of skewing behavior.

We classified the bidders into three groups (small, mid-size and large) on the basis of the number of employees. Small bidders have less than 100 employees, mid-size bidders have between 100 and 300, and large bidders have over 300. We then looked in a variety of ways for systematic differences in the skewing behavior of these groups of bidders. Rather than report specific empirical results, we give a brief summary of our findings. No particular class of bidders was more likely to skew in the right direction, though larger bidders did appear to skew their bids somewhat more dramatically. However, it also appeared that in sales where the mis-estimates were large, smaller bidders were somewhat more likely to win. Overall, we did not find large differences between bid groups.

We also examined the individual skewing behavior of a few frequent participants. None of these were significantly more likely to skew in the right direction, although a few bidders (such as Boise Cascade) skewed more aggressively (that is, the average magnitude of the skew was larger). We might attribute this behavior to a lower level of risk aversion, as Boise Cascade is a large conglomerate.

6 Direct Estimates of the Skewing Model

In the previous section, we tested whether predicted relationships between variables were borne out in the data. We now take a somewhat different cut at the data, by *assuming* that the model is correct and attempting to directly derive and parametrize bidding functions. There are a number of reasons to take this approach: first, if the model is correct, it allows us to quantify the sensitivity of bidders skews to information; and second, it allows us to investigate in some detail the way in which the reserve prices constrain skewing behavior.

6.1 Structural Bid Functions

We start by assuming that bidders have (approximately) CARA utility, so that wealth effects can be ignored. We impose a number of functional form assumptions, beginning with an assumption about the species values $\mathbf{v}$.

(B1) $v_k = r_k + \mathbf{X} \cdot \xi_k + \eta$, for $k = 1, 2$ (so $\Delta v_\chi = \Delta r_\chi$), η independent of $\mathbf{r}, \mathbf{X}, x_\chi, \delta_\chi$.

In an oral auction, the winning bid allocation solves equation (3) from Section 3.2. Writing the information that this bid is conditioned on as $\mathbf{d}_\chi$, we can express the winning skew as $\Delta b_\chi(B, \mathbf{d}_\chi, \mathbf{X}, \mathbf{r}, \varepsilon, x_\chi)$, where

$$\Delta b_\chi(\cdot) - \Delta r_\chi = \begin{cases} \frac{1}{x_\chi}(B - R)/Q_{EST} & \text{if } f(\mathbf{d}_\chi^w, \mathbf{X}, \varepsilon) > \frac{1}{x_\chi}(B - R)/Q_{EST} \\ f(\mathbf{d}_\chi, \mathbf{X}) & \text{otherwise} \end{cases}$$

When the reserve prices do not bind, our assumption that the utility function is CARA implies that the optimal skew $f(\cdot)$ is independent of the total bid. Moreover, Proposition 1 implies that at any equilibrium bid $\Delta b_\chi > \Delta v_\chi = \Delta r_\chi$ so the reserve constraints can only prevent bidders from skewing as much as they want.

The function of interest is the optimal unconstrained skew $f(\cdot)$, which we observe only when the reserve price constraint does not bind. Because whether or not this constraint binds depends on the total bid level B, we need to model this selection. The winning bid level again solves equation (3) in Section 3.2, and can be written as $B(\mathbf{d}_\chi^w, \mathbf{X}, \mathbf{r}, \varepsilon, \eta, x_\chi)$:

$$\frac{B(\cdot) - R}{Q_{est}} = \begin{cases} g^c(\mathbf{d}_\chi^w, \chi, \mathbf{X}, \mathbf{r}, \varepsilon, \eta) & \text{if } f(\mathbf{d}_\chi^w, \chi, \mathbf{X}, \varepsilon) > \frac{1}{x_\chi}(B - R)/Q_{EST} \\ g^u(\mathbf{d}_\chi^w, \chi, \mathbf{X}, \mathbf{r}, \varepsilon, \eta) & \text{otherwise} \end{cases}$$

To complete the model, we make assumptions about functional forms. Under the assumptions of our theoretical model, the bidders observe χ, which provides information about the direction of the optimal skew. As this can be inferred directly from the winner's skew, we order species so that $\Delta b_\chi - \Delta r_\chi \geq 0$. Thus, both the overbid and the skew will always be positive in our model. This motivates a specification of the bidding functions in logarithms (recalling that our continuous control variables are denoted $\mathbf{Z}_i$ and the dummy variables are denoted $\mathbf{W}_i$):

(B2) $\tilde{\delta}_\chi = \ln(\delta_\chi + .3)$; $\tilde{\mathbf{X}}_i = \ln(\mathbf{Z}_i) + \mathbf{W}_i$

(B3) $\tilde{\mathbf{d}}_\chi^w \equiv \ln(\mathbf{d}_\chi^w) = \tilde{\delta}_\chi + \varepsilon$, ε independent of $\mathbf{r}, \mathbf{X}, x_\chi, \delta_\chi$.

(B4) $\ln f(\cdot) = \tilde{\delta}_\chi \alpha_1 + \tilde{\mathbf{X}} \boldsymbol{\alpha}_2 + \varepsilon \alpha_1 \sigma_\varepsilon$.

(B5) $\ln g^{\cdot}(\cdot) = \tilde{\delta}_\chi \beta_1^{\cdot} + \tilde{\mathbf{X}} \boldsymbol{\beta}_2^{\cdot} + \varepsilon \beta_1^{\cdot} \sigma_\varepsilon + \eta \beta_3^{\cdot} \sigma_\eta$.

Consider the interpretation of these equations. First, (B2) introduces logarithmic transformations of the continuous control variables; since our sample includes sales with mis-estimates of as low as $-.27$, we add .3 to the mis-estimate. (B3) states that the bidder's signal about the mis-estimate is the sum of the (transformed) actual mis-estimate, and an independently distributed error term. We interpret ε as the error in the bidder's estimate of δ_χ; a bidder with a higher ε is more optimistic about the extent of the over-estimate on species χ. In (B4) and (B5), we assume that the log of the overbid and the log of the skew are both linear in the log of the signal.

Of course, the functional forms in (B2)-(B5) are *ad hoc*, and we do not attempt to relate them to the primitive utility function; but they offer several distinct advantages. First, below we will require an assumption on the distribution of the errors, and the normal distribution will be convenient; thus, we will assume below that ε and η are normally distributed. Second, although we did not formally model the choice of transformation, several back-of-the-envelope calculations[33] indicated that a logarithmic transformation of the skew and overbid provides a better fit than linearity. Our elasticity estimates are fairly robust to the way in which variables are scaled. Third, and most importantly from a computational perspective, our functional form assumptions imply that the probability that the reserve price binds is a linear function of observables and unobservables:

$$\begin{aligned}
\Pr(\text{res does not bind}) &= \Pr(f(\mathrm{d}^w_\chi, \chi, \mathbf{X}, \varepsilon) < \frac{1}{x_\chi} g^u(\mathrm{d}^w_\chi, \chi, \mathbf{X}, \mathbf{r}, \varepsilon, \eta)) \qquad (11)\\
&= \Pr\left(\begin{array}{c} \tilde{\delta}_\chi \alpha_1 + \tilde{\mathbf{X}}\boldsymbol{\alpha}_2 + \ln(x_\chi) - \tilde{\delta}_\chi \beta^u_1 - \tilde{\mathbf{X}}\boldsymbol{\beta}^u_2 \\ < \varepsilon\beta^u_1\sigma_\varepsilon + \eta\beta^u_3\sigma_\eta - \varepsilon\alpha_1\sigma_\varepsilon \end{array}\right)\\
&\equiv \Pr\left(h(\mathrm{d}^w_\chi, \chi, \mathbf{X}, \mathbf{r}, x_\chi) < \nu\right).
\end{aligned}$$

Notice that x_χ, the fraction of the volume estimated on the over-estimated species χ, affects whether the reserve price binds. To understand its effect, observe that for a small x_χ, only a small overbid is required to achieve the desired level of skew. Thus, in principle, variation in x_χ can be used to identify the full distribution of ν. However, in practice, due to Forest Service bidding restrictions and our focus on sales with two primary species, our data set includes only sales where $x_\chi > .25$. One-fourth of the observations (170 sales) have $x_\chi \in [.25, .44]$; call this group A and the complement group B (524 sales). In group A, the reserve prices bind 30% of the time, as opposed to 45% of the time in group B. This suggests a crude approach to understanding the effects of the reserve prices: estimate the effects of mis-estimates on skews and bids in each group and compare them to results when the sales are divided according to whether the reserve prices were binding in practice. The following table summarizes the results about the effects of mis-estimates:[34]

[33]In particular, we performed Box-Cox estimation on uncensored subsamples of the data, as well as the full sample ignoring censoring; however, that approach is not rigorous unless censoring is accounted for.

[34]These elasticities were estimated using OLS with robust standard errors, and using the specification of control variables as in Table VII.

Elasticities of:	*Contrast: low x_χ (A) v. high x_χ (B)*		*Contrast: Reserve prices binding*	
Skew w.r.t.	Group A	Group B	Not bind	Bind
mis-est. δ_χ	0.065 (0.025)	0.018 (0.010)	0.020 (0.011)	0.032 (0.011)
Overbid w.r.t.	Group A	Group B	Not bind	Bind
mis-est. δ_χ	0.045 (0.021)	0.013 (0.008)	0.010 (0.009)	0.035 (0.011)

The elasticities in this table are all evaluated at the (overall) sample means. Note that there is some ambiguity about interpreting the elasticities with respect to mis-estimates, as we are considering the effect of a larger mis-estimate *in the "right" direction as perceived by the bidder.* In sales where the reserve prices bind (286 sales), bids and skews are more sensitive to mis-estimates than in sales where the reserve prices did not bind. But when censoring is exogenously more likely, bidding and skewing are *less* sensitive to mis-estimates. The latter conforms to our theoretical model: when bidders are unconstrained in their skews, they are able to optimally adjust their skews, in turn allowing them to place higher bids without expecting to pay more. Notice that in uncensored sales, we find only a small and insignificant relationship between the overbid and the mis-estimate. This indicates that in sales where firms chose to skew "moderately," they also chose not to increase the total bid in response to the mis-estimate. On the other hand, when they skewed aggressively, they also increased the total bid sharply in response to higher mis-estimates.

The fact that the way in which the sales are divided affects our comparisons indicates that the censoring is endogenous. Of course, our theoretical model predicts this: the error in the selection equation, ν, is correlated (though imperfectly so) with the errors in the skewing and overbid equations. Formally:

$$E[\ln(\Delta b_\chi(\cdot) - \Delta r_\chi)|\text{res does not bind}] = \tilde{\delta}_\chi \alpha_1 + \tilde{\mathbf{X}}\boldsymbol{\alpha}_2 + \alpha_1 \sigma_\varepsilon E[\varepsilon|\nu > h(\cdot)] \tag{12}$$

$$E[\ln\left(\frac{B(\cdot)-R}{Q_{est}}\right)|\text{res does not bind}] = \tilde{\delta}_\chi \beta_1^u + \tilde{\mathbf{X}}\boldsymbol{\beta}_2^u + E[\varepsilon\beta_1^u\sigma_\varepsilon + \eta\beta_3^u\sigma_\eta|\nu > h(\cdot)] \tag{13}$$

$$E[\ln\left(\frac{B(\cdot)-R}{Q_{est}}\right)|\text{res binds}] = \tilde{\delta}_\chi \beta_1^c + \tilde{\mathbf{X}}\boldsymbol{\beta}_2^c + E[\varepsilon\beta_1^c\sigma_\varepsilon + \eta\beta_3^c\sigma_\eta|\nu < h(\cdot)]. \tag{14}$$

The model fits into the framework of sample selection problems (Heckman, 1976). Of course, the most efficient approach to estimation entails estimating the system of equations simultaneously; with sufficient data, the exogenous variation in x_χ further implies that we could allow for a flexible functional form for the distribution of the errors. However, for computational simplicity and because our dataset is limited, we instead estimate each equation individually using standard single-equation selection routines (for each equation, we used Heckman's two-step procedure, with standard errors computed using full information maximum likelihood). Thus, we assume that η and ε are standard

normal, so that $E[\varepsilon|\nu > h(\cdot)] = \rho_{\varepsilon\nu}\frac{\phi}{1-\Phi}(h(\cdot))$.[35] Also for simplicity as well as robustness, we did not impose the cross-equation restrictions between (11) and the other equations.

Our results from this estimation are reported in Table VII (1) and (2). We use a somewhat more parsimonious specification than in earlier tables. First consider the probit estimates. As expected, higher values of $\ln(x_\chi)$ lead to a lower probability that the reserve prices do not bind. The effect of the mis-estimate is close to zero, although it is somewhat imprecisely estimated. This implies that mis-estimates affect the overbid and the skew at approximately the same rate ($\alpha_1 \approx \beta_1^u$).

Now consider the skewing regression, incorporating the selection correction Table VII (2). Our parameter estimates can be interpreted as elasticities, although we must correct for the transformation of the mis-estimate (this requires multiplying the estimates by $E[\delta_\chi]/(E[\delta_\chi]+.3) = .0461$). The average value of the skew is \$103.50, while the average mis-estimate is .014. We find that a 1% increase in the mis-estimate leads to a .022% increase in the skew. Thus, if the mis-estimate rises by one standard deviation (.075), on average the skew rises by about \$12. Finally, the correlation between the error in the selection and skewing equations is large and negative, at −.80. Recalling the definitions of ν and ε above, if ε and η are independent or positively correlated, this implies that α_1 is large relative to β_1^u. In words, the skew is more sensitive than the overbid to the private signal about the mis-estimate.

The selection model can be usefully contrasted with several alternatives. The first is an ordinary least squares regression on the observations where the reserve price does not bind; the second is a standard censored normal regression model (CNR), which requires that censoring is exogenous. The CNR model estimates:

$$E[\Delta b_\chi - \Delta r_\chi] = \tilde{\delta}_\chi\alpha_1 + \tilde{\mathbf{X}}\boldsymbol{\alpha}_2 + \mathbf{1}_{\{\text{res. binds}\}}\alpha_1\sigma_\varepsilon E[\varepsilon|\tilde{\delta}_\chi\alpha_1 + \tilde{\mathbf{X}}\boldsymbol{\alpha}_2 - \frac{B-R}{Q_{est}\cdot x_\chi} < -\varepsilon]. \tag{15}$$

The results are reported in Table VII (3) and (4). Notice that the coefficients on the mis-estimate from the selection model are greater than the OLS estimates and smaller than the CNR estimates.[36] Intuitively, considering only the uncensored sample leads to a downward bias, as sales where skews are particularly sensitive to mis-estimates may be censored. On the other hand, ignoring the endogeneity of censoring yields an upward bias: the fact that $\frac{B-R}{Q_{est}}$ is positively correlated with both ε and δ_χ leads us to exaggerate the inferred effects of both variables on the desired skew for censored observations. The selection model places a smaller weight on the censoring correction term than the CNR model, leading to more conservative estimates about the effects of ε and δ_χ.

[35] To evaluate the robustness of our results to the normality assumption, we verified that our results do not change when polynomial functions of the Mills ratio $\phi/(1-\Phi)$ are included as explanatory variables. Such an approach serves as a simple first pass at semi-parametric estimation.

[36] We also estimated (but did not report) an ordinary least squares regression on the full sample; the coefficients on the mis-estimate variables were about 2% smaller than the censored normal regression.

Now consider the results for overbids. The elasticity of the overbid with respect to the mis-estimate is approximately .01 and significantly different from 0 only at the 15% confidence level. Accounting for selection (as opposed to (unreported) OLS or CNR) lowers the estimated effects of δ_χ. On the other hand, we find larger and significant effects of mis-estimates on overbid for censored sales; the elasticity is approximately .03. This finding runs counter to the following intuition: we would expect that when the bidders are constrained by the reserve price, they will not be able to skew as effectively and efficiently, and thus their bids will be less sensitive to mis-estimates. One way to reconcile our results with the intuition is to consider the possibility that the econometric model does not fully account for the heterogeneity across sales. For example, the sales where censoring occurs and bids are high, might be sales where the bidders are more confident about the direction of the mis-estimate. Another hypothesis is that our functional form does not fully capture nonlinear effects of the mis-estimate.

Recalling our analysis from Section 5.3, we now use our estimates to calculate the effect of mis-estimates on revenue. Equation (9) indicates that an increase in δ_χ affects the magnitude of the bid as well as the gap between bid and payment, $\Delta b_\chi \cdot \delta_\chi$. In contrast to Section 5.3, we now examine the effect of an increase in the mis-estimate in the direction of χ. In our sample, the average predicted Δb_χ is quite large (107.8), so that an increase in the mis-estimate leads to a large increase in the gap between the bid and the payment. We find that for uncensored sales, the elasticity of revenue with respect to mis-estimates is -.0066; thus, an increase in the mis-estimate of one standard deviation (.075) leads to a reduction in revenue of \$4.74 per Mbf., or about \$14,560 on the average sale. In sales where the reserve prices bind, the bid increases at a rate proportional to the skew. For these (less frequent) sales, we find that revenue actually increases when the mis-estimate grows; the elasticity is .0032.

Overall, the magnitudes of our estimates in this section should still be interpreted with caution, due to the fairly strong functional form assumptions. Nonetheless, our results may provide some insight into the direction of, and relative importance of, the effects of the reserve prices; as well, they indicate that skewing leads to small but statistically significant reductions in revenue.

7 Conclusion

In this paper, we have considered the effects of information on bidding behavior in Forest Service timber auctions. The rules of the "scale sales" create incentives for bidders to distort their bids. By using ex post information about the value of the tract, we are able to provide evidence that bidders are risk-averse, have private information about the underlying characteristics of the tract, and exploit this information in their bidding behavior. This information appears to play a role in allocating the tract between bidders, since bidders who take larger gambles are more likely to

win the auction. Further, larger realizations of the mis-estimate typically lead to lower revenue for the Forest Service, consistent with the hypothesis that the bidders require a risk premium to compensate for the large gambles they must take to remain competitive in the auction.

Our use of ex post information has an additional advantage: our results provide evidence about the validity of several of the assumptions commonly invoked in auction studies. The hypothesis of pure private values, which requires that all private information on the part of a given bidder concerns the bidder's own private costs and benefits, is not supported by our analysis. Observed bidding behavior is also inconsistent with risk-neutrality. Finally, bidders clearly expend resources to obtain strategically useful information, suggesting that attention should be given to the process of information acquisition.

In future work, it may be possible to use the ex post value information in a more complete structural model. For example, we might be able to assess the relative salience of heterogeneous values for the tract and private signals about mis-estimates. Alternatively, we could explore direct estimates of bidder risk-aversion. We might also quantify the extent of the winner's curse in this market, and the value of additional information. However, these questions would require a number of demanding extensions to the theoretical and econometric models.

8 Appendix A: Proofs

The following results are used in the formal analysis. See the appendix of Milgrom and Weber (1982) for basic results about affiliation and Athey (1998) for a unified treatment of these lemmas and further references. For our purposes, we need the following definitions. $g : \Re^2 \rightarrow \Re$ is *bivariate single crossing* in $(x; y)$ (see Milgrom and Shannon, 1994) if for all $x^H > x^L$ and $y^H > y^L$ $g(x^H, y^L) - g(x^L, y^L) \geq (>) 0$ implies $g(x^H, y^H) - g(x^L, y^H) \geq (>) 0$. A function f is *supermodular* if, for all $y^H > y^L$, $f(y^H, z) - f(y^L, z)$ is nondecreasing in z; f is strictly supermodular if the difference is increasing. If f is positive, f is (strictly) *log-supermodular* if $\log(f)$ is (strictly) supermodular. If y and z are random variables with positive joint density $f(y, z)$ with respect to Lebesgue measure, y and z are (strictly) *affiliated* if and only if f is (strictly) *log-supermodular* in (y, z) almost everywhere (Lebesgue measure). A vector of random variables $\mathbf{z}$ with a positive joint density $f(\mathbf{z})$ is (strictly) affiliated if $f(\mathbf{z})$ is (strictly) log-supermodular in (z_i, z_j) almost everywhere, for all $i \neq j$.

Lemma 7 *(Milgrom and Shannon, 1994) If $g(x, \theta)$ satisfies bivariate single crossing, $x^*(\theta) = \arg\max_x g(x, \theta)$ is nondecreasing in θ.*[37]

[37] If there are multiple optima or no optima for some parameter values, the set of optimizers is nondecreasing in the strong set order (Milgrom and Shannon, 1994).

Lemma 8 *Let* (y,z) *be affiliated random variables with conditional density* $f(y|z)$. *If* $g(x,y,z)$ *is bivariate single crossing in* $(x;y)$ *and supermodular in* (x,z), *then* $\int g(x,y,z)f(y|z)dy$ *is bivariate single crossing in* $(x;z)$.

Lemma 9 *Let* $u(w;\theta)$ *be differentiable and increasing in* w, *and suppose that* θ *decreases the coefficient of absolute risk aversion of* $u(w;\theta)$ *(*$u_w(w;\theta)$ *is log-supermodular). Then* $\int u(x\cdot y;\theta)f(y)dy$ *is bivariate single crossing in* $(x;\theta)$.

Lemma 10 *If* (y,z) *are strictly affiliated, and* $g(y)$ *is increasing, then* $E[g(y)|z]$ *is increasing in* z.

Lemma 11 *Suppose that* $(y,\mathbf{z})$ *are strictly affiliated. (i) If* $g(y)$ *is increasing, then* $E[g(y)|\ z_i \in [a_i,b_i], i=1,..,n]$ *is increasing in* a_i *and* b_i *for all* i. *(ii) If* $g(x,y)$ *is bivariate single crossing in* $(x;y)$, *then* $E[g(x,y)|\ z_i \in [a_i,b_i], i=1,..,n]$ *is bivariate single crossing in* $(x;a_i)$ *and* $(x;b_i)$.

Lemma 12 *If* (y,z) *are affiliated, and* $g(x,y)$ *is supermodular, then* $\int g(x,y)f(y|z)dy$ *is supermodular in* (x,z).

Proof of Proposition 2. The bid allocation problem can be written as:

$$\max_{\Delta b_\chi} E_{\delta_\chi}\left[u\left(Q_{ACT}\cdot\left((\Delta b_\chi-\Delta v_\chi)\cdot\delta_\chi+\mathbf{v}\cdot\mathbf{x}-\frac{B^j}{Q_{EST}}\right)\right)\mid d_\chi^j,\chi;\ \forall k\neq j,\ B^k(d_\chi^k)<B^j\right]$$
$$\text{subject to:}\qquad b_1\geq r_1,\ b_2\geq r_2,$$

Let $U(\Delta b_\chi;B^j,d_\chi^j)$ represent the objective in this optimization program. Fix B^j. Then bidder j's payoff function, $u(\cdot)$ is bivariate single crossing in $(\Delta b_\chi,\delta_\chi)$, i.e. $\frac{\partial u}{\partial\Delta b_\chi}$ crosses zero once from below as a function of δ_χ. Moreover, from *(A1)*, $(\delta_\chi,d_\chi^j,d_\chi^{-j})$ are affiliated. Thus, by Lemma 8, for fixed B^j, $U(\Delta b_\chi;B^j,d_\chi^j)$ is bivariate single crossing in $(\Delta b_\chi,d_\chi^j)$. The monotonicity theorem of Milgrom and Shannon (1994) (Lemma 7) then implies that $\Delta b_\chi^j(B^j,d_\chi^j)$ is nondecreasing in d_χ^j for a fixed B^j. Now fix d_χ^j. An increase in B^j has two effects. First, it acts as a downward movement in wealth, which, under CARA or IARA, makes the bidder no more risk-averse and increases Δb_χ^j (Lemma 9). Second, it increases the set of opponent types defeated; by Lemma 11 this effect favors an increase in Δb_χ^j. So $\Delta b_\chi^j(B^j,d_\chi^j)$ will be nondecreasing in B^j for a fixed d_χ^j. *Q.E.D.*

Proof of Proposition 3. *(i)* First, notice that by bidding $B=V$ and $\Delta b_\chi=\Delta v_\chi$ (i.e. $b_i=v_i$), bidder j can ensure a payoff of zero. If $\beta(\cdot)\geq V$, then any total bid $B<V$ also results in a payoff of zero, so bidder j can restrict attention to bids that are greater than or equal to V. We now establish monotonicity. Define $m_\chi^{-j}=\max_{l\neq j}\{d_\chi^l\}$. Since the signals were assumed to be exchangeable, m_χ^{-j} is affiliated with d_χ^j (Milgrom and Weber, 1982). Given an optimal choice of Δb_χ^j, the bidder chooses B to maximize expected utility. In order to analyze the problem, it will be convenient to break the interaction between B and d_χ^j into two components, the direct effect of

d^j_χ on payoffs for a fixed Δb^j_χ, and the indirect effect arising due to the fact that the optimal choice of Δb^j_χ depends on d^j_χ. To separate the two effects, we write payoffs as

$$\overline{U}(B, d^j_\chi, y) = U(\Delta b^j_\chi(B,y), B, d^j_\chi) \cdot \int_{[0,1]^{I-1}} \mathbf{1}_{\{\beta(m_\chi^{-j})<B\}}(m_\chi^{-j}) dF(m_\chi^{-j}|\chi, d^j_\chi),$$

and show that $\overline{U}(B, d^j_\chi, d^j_\chi)$ is bivariate single crossing in $(B; d^j_\chi)$, which by Lemma 7 will imply our result. To do so, we show that $\frac{\partial}{\partial B}\overline{U}(B, d^j_\chi, y)$ is single crossing in d^j_χ and y at $y = d^j_\chi$.

To begin, consider the effect of y on $\frac{\partial}{\partial B}\overline{U}(B, d^j_\chi, y)$. Letting subscripts denote partial derivatives (and applying the envelope theorem in computing $\frac{\partial}{\partial B}\overline{U}(B, d^j_\chi, y)$), we have

$$\frac{\partial^2}{\partial B \partial y}\overline{U}(B, d^j_\chi, y) = U_1(\Delta b^j_\chi(B,y), B, d^j_\chi)\frac{\partial}{\partial y}\Delta b^j_\chi(B,y) f(\beta^{-1}(B)|\chi, d^j_\chi)$$

$$+U_{12}(\Delta b^j_\chi(B,y), B, d^j_\chi)\frac{\partial}{\partial y}\Delta b^j_\chi(B,y) \cdot \int_{[0,1]^{I-1}} \mathbf{1}_{\{\beta(m_\chi^{-j})<B\}}(m_\chi^{-j}) dF(m_\chi^{-j}|\chi, d^j_\chi).$$

Applying the envelope theorem again, we know that $U_1(\Delta b^j_\chi(B,y), B, d^j_\chi) = 0$ at $y = d^j_\chi$. Further, our arguments in Proposition 2 imply that $U_{12}(\Delta b^j_\chi(B,y), B, d^j_\chi) \geq 0$ at $y = d^j_\chi$, using the assumption of CARA/IARA and Lemma 10. Since $\frac{\partial}{\partial y}\Delta b^j_\chi(B,y) \geq 0$ by Proposition 2, it follows that $\frac{\partial^2}{\partial B \partial y}\overline{U}(B, d^j_\chi, y) \geq 0$ at $y = d^j_\chi$.

Now consider the effect of d^j_χ on $\frac{\partial}{\partial B}\overline{U}(B, d^j_\chi, y)$ when y is fixed. We introduce some additional notation:

$$\hat{U}(B, \Delta b^j_\chi, d^j_\chi, m_\chi^{-j}) = \int_{[0,1]} u\left(\pi\left(\Delta b^j_\chi, B, \delta_\chi\right)\right) dF(\delta_\chi | d^j_\chi, m_\chi^{-j}, \chi),$$

so that

$$\overline{U}(B, d^j_\chi, y) = \int_{[0,1]^{I-1}} \hat{U}(B, \Delta b^j_\chi(B,y), d^j_\chi, m_\chi^{-j}) \cdot \mathbf{1}_{\{\beta(m_\chi^{-j})<B\}}(m_\chi^{-j}) dF(m_\chi^{-j}|\chi, d^j_\chi).$$

By the envelope theorem, we can ignore the effect of B on $\hat{U}$ through Δb^j_χ when computing $\frac{\partial}{\partial B}\overline{U}(B, d^j_\chi, y)$. To apply Lemma 8, we need to show that $\hat{U}$ is supermodular in (B, d^j_χ) and (B, m_χ^{-j}). This will in turn imply that the integrand in $\overline{U}(B, d^j_\chi, y)$ is supermodular in $(B; d^j_\chi)$ and bivariate single crossing in $(B; m_\chi^{-j})$. To establish the desired properties of $\hat{U}$, we compute $\frac{\partial}{\partial B}u\left(\pi\left(\Delta b^j_\chi, B, \delta_\chi\right)\right) = -\frac{Q_{ACT}}{Q_{EST}}u'(\pi)$. Recall from Proposition 1 that $\Delta b^j_\chi(B, d^j_\chi) > \Delta v_\chi$, and thus $\pi\left(\Delta b^j_\chi, B, \delta_\chi\right)$ is nondecreasing in δ_χ. Since u is concave, it follows that $\frac{\partial}{\partial B}u$ is nondecreasing in δ_χ. Then, since δ_χ and m_χ^{-j} are affiliated, Lemma 12 implies that $\hat{U}$ is supermodular in $(B; m_\chi^{-j})$. The argument for $(B; d^j_\chi)$ is analogous.

(ii) The above argument verifies the single crossing condition in Athey's (1997) existence theorem for the case of two bidders. The result can be applied to the case of J symmetric bidders by

looking for a fixed point in the best response correspondence for a single bidder, when all opponents play the same nondecreasing strategy. *Q.E.D.*

Proof of Proposition 4: Suppose to the contrary that $R < B^*(D;k,P_k) < V$. Then there exist some b_1, b_2 such that $Q_{EST}(\mathbf{b} \cdot \mathbf{x}) = B^*(D;k,P_k)$ and $r_i \leq b_i \leq v_i$. This "safe" portfolio has strictly positive expected profits for any resolution of uncertainty, contradicting the fact that the optimal skew gives zero expected utility. So $B^*(\cdot) \geq V$. Now, by Proposition 1, at the optimum, $\Delta b_\chi^*(D;k,P_k) > \Delta v_\chi$, which implies that $b_\chi^* \geq v_\chi > r_\chi$. Let $\pi(\Delta b, B, \delta) = Q_{ACT} \cdot ((\Delta b - \Delta v)\delta + (V - B)/Q_{EST})$ and let λ_χ and $\lambda_{\neg\chi}$ denote the Lagrange multipliers on the reserve price constraints. We have

$$\begin{aligned} 0 \quad = \quad & E_{\delta_\chi}\left[u\left(\pi(\Delta b_\chi^*(D;k,P_k), B^*(D;k,P_k), \delta_\chi)\right) \mid d_\chi^1 = \ldots = d_\chi^{J-k} = D, P_k, \chi\right] + \\ & + \lambda_\chi\left[(B/Q_{EST}) + \Delta b_\chi(1 - x_1) - r_1\right] + \lambda_{\neg\chi}\left[(B/Q_{EST}) - \Delta b_\chi x_1 - r_2\cdot\right]. \end{aligned} \tag{16}$$

Since $\Delta b_\chi^* > \Delta v_\chi$, π is increasing in δ_χ. Lemma 11 implies the right-hand side of (16) is increasing in D; it is decreasing in B since π is decreasing in B and $\lambda_{\neg\chi} \geq 0 = \lambda_\chi$. The result follows. *Q.E.D.*

Proof of Proposition 5. *(i)* Suppose bidders $l \neq j$ follow the strategies described in the Proposition. Then, if all opponents are still active at a bid B, all that can be inferred is that $d_\chi^l \geq D^*(B;k,P_k)$ for each opponent l. Suppose no one has dropped out; we will consider player j's incentives to deviate from the equilibrium strategy. At $B = B^*(d_\chi^j;0,\emptyset)$, bidder j knows that he will win at B only if all opponent's signals are equal to $D^*(B;0,\emptyset) = d_\chi^j$. For him to earn non-negative expected profits if he wins, he must announce $\Delta b^E(B;0,\emptyset)$. So bidder j has no incentive to deviate at $B = B^*(d_\chi^1;0,\emptyset)$. Moreover, as B rises past $B^*(d_\chi^j;0,\emptyset)$, j can not possibly make positive expected profits if he wins. So he will drop out just beyond $B^*(d_\chi^j;0,\emptyset)$.

Now consider j's bid allocation announcement Δb^j when $B < B^*(d_\chi^j;0,\emptyset)$. Consider first an announcement $\Delta b < \Delta b^E(B;0,\emptyset)$. Suppose that following such a deviation, opponent's beliefs are the same as if j announced $\Delta b^E(B;0,\emptyset)$, that is, opponents do not update at all. Then the deviation will be payoff-relevant only if bidder j actually wins at B, i.e. if $d_\chi^l = D^*(B;0,\emptyset)$ for all. Given this event, we know that if j had signal $D^*(B;0,\emptyset)$, he would prefer a skew of $\Delta b^E(B;0,\emptyset)$ to any lower Δb. But in fact, $d_\chi^j > D^*(B;0,\emptyset)$, so j certainly prefers a skew of $\Delta b^E(B;0,\emptyset)$ to any lower Δb (this follows from the fact that the objective defining Δb^E is bivariate single crossing in Δb and d_χ^1). So this deviation is not profitable.

Finally, suppose $B < B^*(d_\chi^j;0,\emptyset)$ and consider a deviation to $\Delta b > \Delta b^E(B;0,\emptyset)$. Note that if $d_\chi^j < D^*(B;0,\emptyset)$, then j's expected profits from bidding $\Delta b > \Delta b^E(B;0,\emptyset)$ are negative. Since opponents already believe that $d_\chi^j \geq D^*(B;0,\emptyset)$, it makes sense to assume that if they do update their beliefs they will revise upward. To this end, assume that any opponent l who is marginal,

i.e. who plans to drop out immediately, revises his beliefs about j to be that $d_\chi^j \geq D^+(B;0,\emptyset) > D^*(B;0,\emptyset)$, where $D^+(B;0,\emptyset)$ is large enough to deter l from dropping out. All other bidders make no revision in their beliefs about d_χ^j. With these off-path beliefs, a bid allocation deviation by bidder j to $\Delta b > \Delta b^E(B;0,\emptyset)$ cannot increase and may decrease j's expected payoffs (since opponents have more optimistic beliefs about his signal and will stay in the auction longer). The same arguments can be applied after any number of bidders have dropped out, which ensures that the proposed strategies form an equilibrium. Note that a variety of other off-equilibrium-path belief restrictions would also suffice. So long as in the event of a bid allocation deviation by some bidder j, opponents are at least as optimistic about his signal as in the equilibrium we have described, he will have no incentive to deviate.

(ii) The proposed equilibrium fully reveals the signals of all players except the player with the highest signal. *Q.E.D.*

Proof of Proposition 6: The result follows because nondecreasing functions of affiliated random variables are affiliated. In the sealed bid auction equilibrium of Proposition 3, $\Delta b_\chi^j(B, d_\chi)$ is nondecreasing in both arguments, and $B^j(d_\chi)$ is nondecreasing, so that $\Delta b_\chi^j(B^j(d_\chi), d_\chi)$ is nondecreasing in d_χ. Since d_χ and δ_χ are affiliated, so are Δb_χ^j and δ_χ (and similarly for a fixed bid B). Now consider the oral auction equilibrium described in Proposition 5. Each bidder fully reveals their signal on dropping out. Suppose k bidders have dropped out. Then the next observed drop-out point B and skew Δb_χ will be higher, the higher is the signal of each of the lowest $k+1$ signals. Since each of these signals is affiliated with δ_χ, Δb_χ will be affiliated with δ_χ. Finally, if $\chi = 1$, then $\delta = \delta_\chi$ and $\Delta b = \Delta b_\chi$ so δ and Δb are affiliated conditional on $\chi = 1$. If $\chi = 2$, then, an increase in $-d_\chi$ increases Δb and d. Since, conditional on $\chi = 2$, δ and $-d_\chi$ are affiliated, so are δ and Δb. Thus δ and Δb are affiliated. *Q.E.D.*

9 Appendix B: The Data

In this section, we briefly describe our data sources and criteria for selecting our sample. The data can be divided into two categories: "bidding data" and "cutting data." The bidding data contains sale appraisal information as well as the highest bids placed by each bidder at the auction. This data is publicly available from the Forest Service immediately after the auction. The cutting data is the ex post information about the timber actually removed from the tract. This data is also publicly available from the Forest Service. Because the Forest Service uses different coding systems for the different species of timber in the bidding and cutting data, the Forest Service data is somewhat difficult to work with, and so we purchased "matched" data from one of the leading industry data sources, Timber Data Company.

We selected a number of forests, focusing on larger forests that had a large fraction of sales with two major species. The forests we obtained include Region 6, Forests 1-6, 9-12, and 15-18; Region 5, Forests 3, 5, 6, 10, 11, and 14-18. Among the sales from these forests, we consider only sales where the matching process met certain reliability criteria. We then narrowed the sample along a number of criteria.

In Region 6, we ruled out sales where the average bid per Mbf. was within $1 of the reservation price, as such sales leave little scope for skewing; similarly, we ruled out sales where the overbid per Mbf. was less than 5% of the appraised difference in values, Δr. We further ruled out sales with extreme mis-estimates (greater than 27%) or gaps between aggregate volume sold and cut (greater than 30% in magnitude on the sale, or 25% on either species), as these sales may have special circumstances (for example, cutting may have been aborted for some reason). We then dropped outliers along a number of dimensions, including only sales with volume estimated between 100 Mbf. and 25000 Mbf., and density less than 115 Mbf./acre. Finally, we dropped sales where road construction was greater than 2.5% of the value of the sale. Since the government reimburses road construction using a complicated system of credits, bids in sales with low road construction can be interpreted more directly in terms of expected payments. For Region 5, we used essentially the same criteria, with one notable exception: we allowed sales with road construction valued at up to 100% of the appraised value of the tract. This allows a larger sample size, a problem for Region 5 sealed-bid auctions.

Finally, we note a subtlety that arises in interpreting the bidding data. Forest Service regulations require that no matter what the appraised values and costs, all per-unit bids must clear a certain minimum value, known as the "base rate." On a small number of sales, the base rate is greater than the reserve price. Thus, the Forest Service might accept a bid that violates the base rate rule. In such a case, the Forest Service uses a pre-announced mechanism to lower the bid on some species and raise it on others. In this way, firms can sometimes achieve skews that violate the posted reserve price constraints. Fortunately, the Forest Service data includes the information required for us to compute the amount a bidder anticipates paying when the bid is placed. This is known as the "statistical bid," and all of our results use the statistical bids. In any case, only a few sales are affected by this rule.

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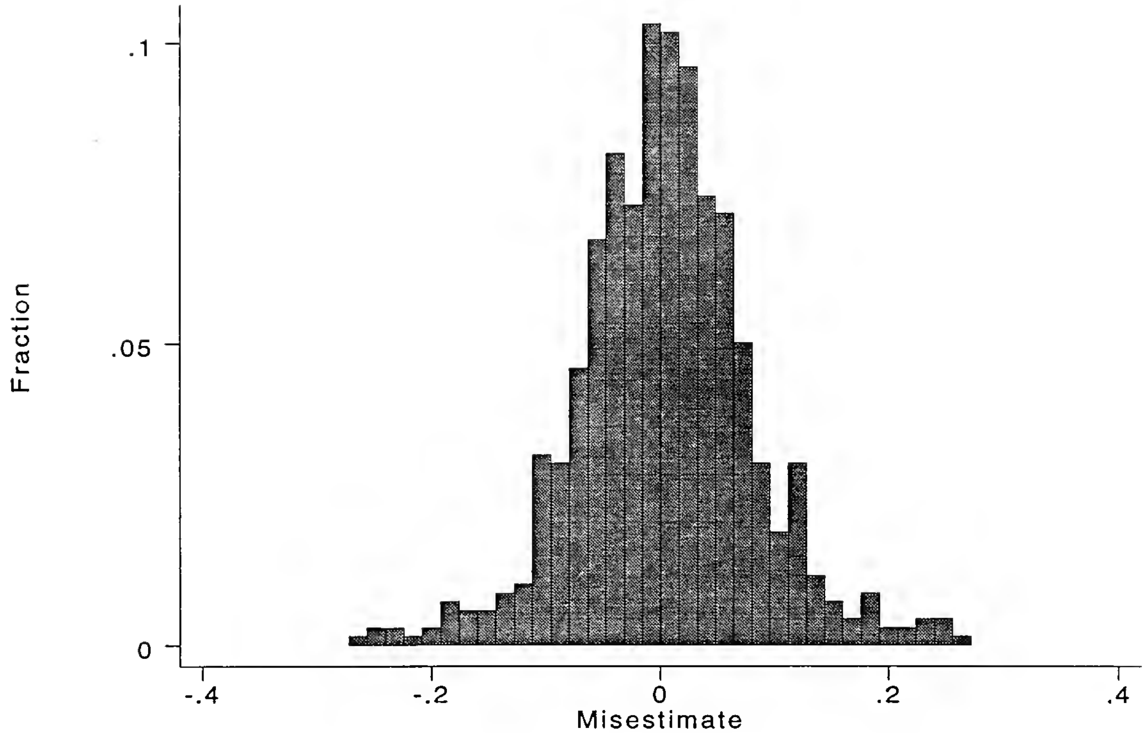


Chart I: Histogram of mis-estimates, species 1 is the high-valued species.
Sample of 699 Region 6 oral auctions.

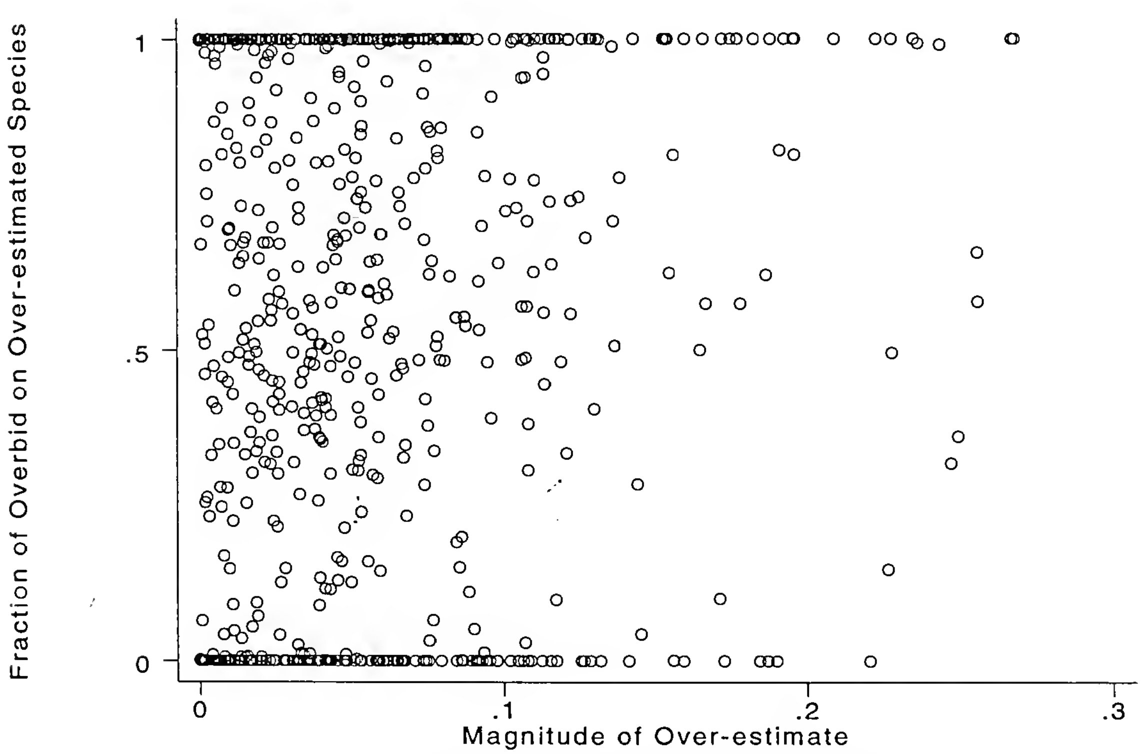


Chart II: Percentage of overbid placed on the over-estimated species against the over-estimate. Sample of 699 Region 6 oral auctions.

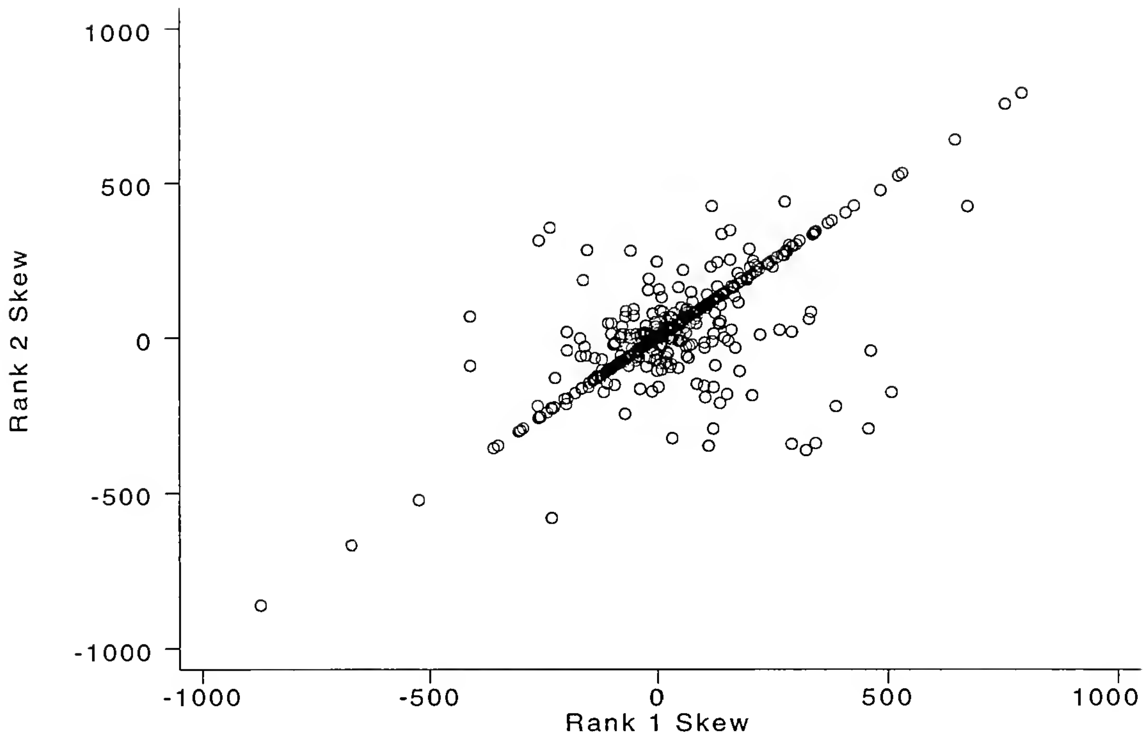


Chart IIIA: Plot of Rank 2 Skew Against Rank 1 Skew (onto over-estimated species); Sample of Region 6 oral auctions with 3 or more bidders

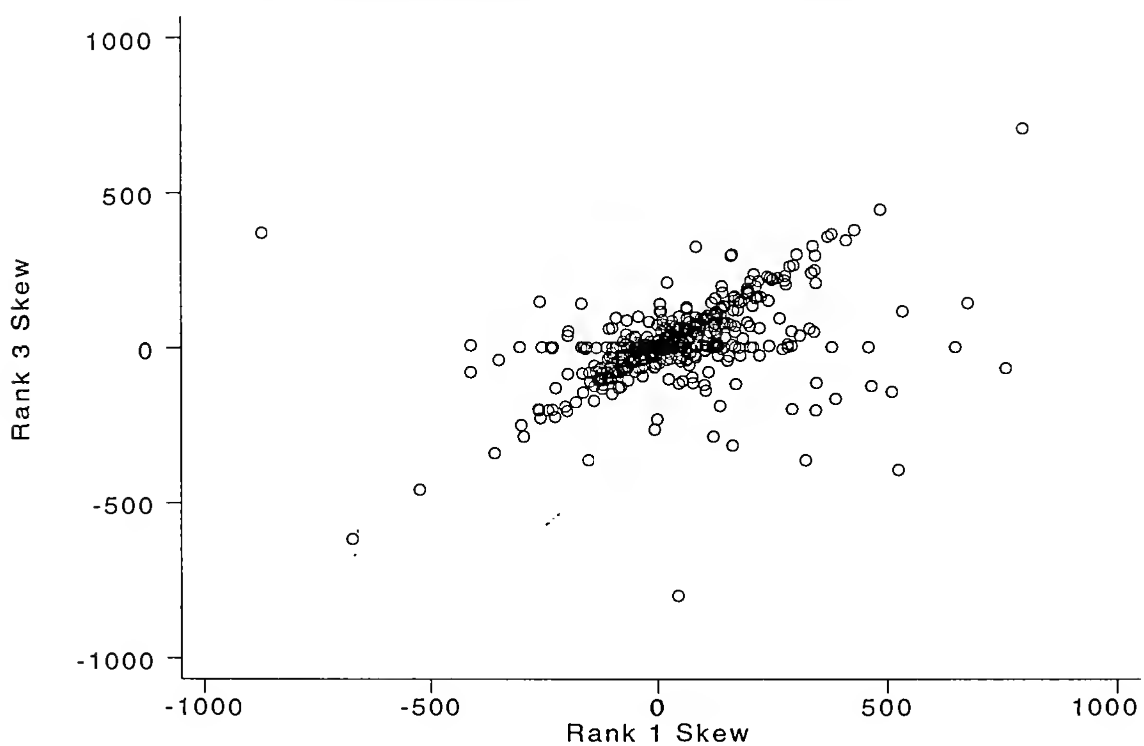
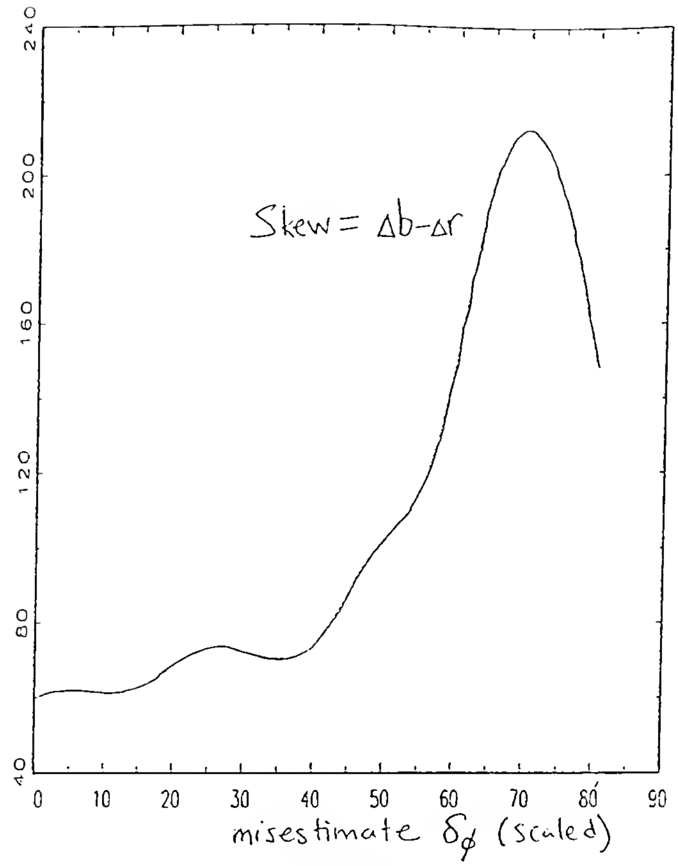


Chart IIIB: Plot of Rank 3 Skew Against Rank 1 Skew (onto over-estimated species); Sample of Region 6 oral auctions with 3 or more bidders



KERNEL DENSITY

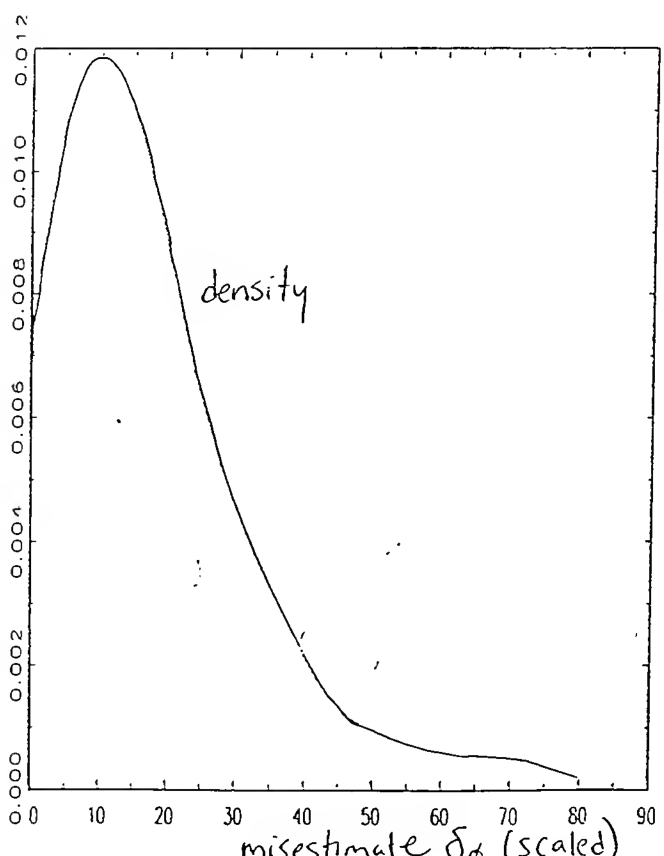


Chart IV: Kernel regression.

Table I: Summary Statistics for Oral and Sealed Auctions with Two Primary Species				
	Region 6 Oral		Region 5 Sealed	
N	*699*		*63*	
	Mean	**Std. Dev.**	**Mean**	**Std. Dev.**
Bidding and Volume Variables				
s1 bid rate = b1	166.06	138.99	118.38	108.88
s2 bid rate = b2	135.34	116.42	106.00	111.42
s1 reserve rate = r1	76.43	73.65	53.40	56.37
s2 reserve rate = r2	76.54	73.65	35.86	35.83
s1 market value	439.90	98.16	344.18	108.27
s2 market value	438.42	88.87	309.30	97.22
Volume est. (mmbf)	3.25	3.71	1.70	3.37
Volume cut (mmbf)	3.02	3.35	1.85	3.80
Average reserve price (per mbf)	74.95	55.69	43.70	33.23
Estimated % of Volume on high-valued species	0.520	0.125	0.543	0.126
Actual % of Volume on high-valued species	0.517	0.138	0.529	0.156
Tot. Bid (B)	467380.10	620525.50	247820.40	682193.00
Tot. Paid (P)	426688.40	566565.30	259778.20	741056.60
(Tot. Bid)/(Volume Est.) = B/Qest	142.98	83.44	104.85	56.73
(Tot. Paid)/(Volume Cut) = P/Qcut	139.07	81.23	99.91	52.53
Skewing Variables				
% of (Tot. Bid - Res. Price) on over-est. species	0.57	0.40	0.49	0.30
Skew onto over-est species: Δb-Δr	30.79	150.88	-5.35	139.15
Revenue Shortfall from Skew	10581.69	57819.42	9780.05	28956.45
Revenue Shortfall from Undercut	30109.96	110740.90	-21737.85	88802.78
Misestimate Variables				
(Volume Cut–Volume Est)/(Volume Est.)	-0.066	0.157	0.036	0.125
Misest. on s1=high-valued sp: Est. % s1 - Act. % s1	0.003	0.076	0.015	0.072
Absolute magnitude of mis-estimate	0.057	0.051	0.049	0.054
Bidder Participation Variables				
# of bidders	6.06	3.12	5.32	2.60
SBA dummy	0.21	0.41	0.17	0.38
Sale Characteristics				
Contract Length/Volume	0.01	0.02	0.02	0.02
Density of Timber (mbf/acre)	27.98	24.31	12.32	14.74
Volume per-acre material per mmbf	80.87	168.29	0.00	0.00
Logging costs est. (thousands of $)	401.05	566.08	159.34	213.17
Road construction (thousands of $)	0.61	3.02	11.21	31.12

Table II: Probability of Skewing in the Right Direction

Dependent Variable **: Dummy = 1 if Skew*Misestimate =(Δb-Δr)*$\delta 1$ >0**

	(1)		(2)	
	Probits			
	coefficient	**Robust s.e.**	**coefficient**	**Robust s.e.**
Misestimate Variables				
δ1 = misestimate	3.1378	(1.0884)	6.7987	(2.6894)
δ1^2 (misestimate squared)			-19.5529	(13.0112)
Volume and Reserve Price Controls				
Volume est. (mmbf)	-0.0169	(0.0448)	-0.01831	(0.04525)
Volume squared	0.00387	(0.00276)	0.00386	(0.00279)
Average reserve price per mbf	-0.00039	(0.00124)	-0.00041	(0.00125)
Bidder Participation				
# bidders	0.0286	(0.0194)	0.0313	(0.0196)
SBA sale (dummy)	-0.1189	(0.1302)	-0.1213	(0.1304)
Other Sale Characteristics				
Contract Length/Volume	4.7634	(3.5466)	4.8589	(3.5802)
Density of Timber	0.00339	(0.00295)	0.00352	(0.00295)
Volume per-acre material	-0.00041	(0.00034)	-0.00041	(0.00034)
Logging costs est. per mmbf	-0.00014	(0.00021)	-0.00014	(0.00021)
constant	0.1064	(0.3856)	0.0110	(0.3960)
	N =699		N = 699	
	Chi-sq(30)	44.37	Chi-sq(31)	46.5
	Prob>Chi-sq	0.0441	Prob>Chi-sq	0.0364
	Psuedo R-sq	0.0496	Psuedo R-sq	0.0519

Notes: Species, Forest, and Year dummies included in each specification.

Sample includes Region 6 oral auctions. Species ordered so that sp. 1 is over-estimated.

Table III: Skewing in Response to Misestimates

Dependent Variable **: Skew = Δb-Δr = (s1 bid rate - s2 bid rate) -(s1 reserve - s2 reserve)**

	(1) OLS		(2) OLS		(3) OLS	
	coefficient	robust s.e.	coefficient	robust s.e.	coefficient	robust s.e.
Total Bid/Volume Est.						
Total Bid/Volume Est. = B/Qest					0.793	(0.277)
Misestimate Variables						
$\delta 1$ = misestimate			383.751	(140.117)	319.617	(121.855)
Volume and Reserve Price Controls						
Volume est. (mmbf)	-2.206	(4.37)	-1.257	(4.526)	-1.512	(4.401)
Volume squared	0.461	(0.302)	0.423	(0.308)	0.438	(0.297)
Average reserve price per mbf	0.081	(0.174)	0.105	(0.174)	-0.753	(0.299)
Bidder Participation						
# bidders	4.115	(2.48)	4.061	(2.444)	-0.794	(2.610)
SBA sale (dummy)	-12.677	(15.316)	-14.568	(15.284)	-15.317	(15.474)
Other Sale Characteristics						
Contract Length/Volume	79.607	(469.510)	52.235	(479.194)	34.857	(494.604)
Density of Timber	0.511	(0.381)	0.534	(0.381)	0.309	(0.374)
Volume per-acre material	-0.029	(0.033)	-0.031	(0.033)	-0.026	(0.032)
Logging costs est. per mmbf	-0.030	(0.021)	-0.030	(0.022)	-0.029	(0.023)
constant	42.4	(54.4)	27.9	(55.3)	-16.18	(58.72)
	N = 699		N = 699		N = 699	
	F(29, 669)	1.41	F(30, 668)	1.51	F(30, 668)	1.95
	Prob > F	0.0759	Prob > F	0.0416	Prob > F	0.0017
	R-squared	0.0604	R-squared	0.0754	R-squared	0.116

Notes: Species, Forest, and Year dummies included in each specification.

Sample includes Region 6 oral auctions. Species ordered so that sp. 1 is over-estimated.

Table IV: Skewing and Skewing Outcomes, All Ranks

	(1)		(2)		(3)	
			Fixed Effect Regressions			
Dependent Variable:	**Dummy=1 if Skew in Right Direction**		**Δb-Δr Skew**		**$\|\Delta b$-$\Delta r\|$ Magnitude of skew**	
	coefficient	**s.e.**	**coefficient**	**s.e.**	**coefficient**	**s.e.**
Rank Dummies						
rank2	2.04E-10	(0.0296)	4.8150	(12.8003)	-2.0061	(9.6092)
rank3	-0.0365	(0.0299)	-24.6335	(12.8003)	-19.6113	(9.6092)
rank4	-0.0776	(0.0299)	-34.3153	(12.8003)	-46.3047	(9.6092)
constant	0.5799	(0.0211)	41.2283	(9.0512)	87.8910	(6.7947)
sale	F(218,654) = 7.139		F(71,213) = 5.320		F(71,213) = 6.471	
	N=876	**n=219**	**N=288**	**n=72**	**N=288**	**n=72**
	corr(u_key, Xb) = 0.0000		corr(u_testn, Xb) = -0.0000		corr(u_testn, Xb) = -0.0000	
	F(3, 654) = 3.07		F(3, 213) = 4.37		F(3, 213) = 9.96	
	Prob>F = 0.0274		Prob > F = 0.0052		Prob>F = 0.0000	

Notes: Sample includes Region 6 oral auctions. Species ordered so that sp. 1 is over-estimated. Specification (1) includes all auctions with at least four bidders.

Table V: Reduced-Form Revenue Effects of Misestimates

Dependent Variable	(1) **Total Bid/Volume Est - Total Paid/Volume Cut**		(2) **Total Bid/Volume Est**		(3) **Total Paid/Volume Cut**	
					OLS Regressions	
	coefficient	**Robust s.e.**	**coefficient**	**Robust s.e.**	**coefficient**	**Robust s.e.**
Total Bid/Volume Est.						
Total Bid/Volume Est. = B/Qest						
Misestimate Variables						
δ1*dummy(Δr<0) = mis-est. on low-valued species	110.26	(34.01)	171.10	(59.40)	60.84	(43.29)
δ1*dummy(Δr>=0) = mis-est. on high-valued species	81.63	(23.81)	1.19	(46.92)	-80.44	(43.63)
Volume and Reserve Price Controls						
Ave. reserve price using actual volumes	-0.616	(0.200)	-0.455	(0.278)	0.161	(0.230)
Ave. reserve price using estimated volumes	0.633	(0.200)	1.537	(0.284)	0.904	(0.242)
Volume est. (mmbf)	-0.012	(0.431)	2.419	(1.266)	2.430	(1.245)
Volume squared	0.017	(0.027)	-0.108	(0.090)	-0.125	(0.088)
Bidder Participation						
SBA sale (dummy)	-0.50	(1.31)	2.90	(4.52)	3.40	(4.78)
Other Sale Characteristics						
Contract Length/Volume	-19.7	(50.1)	-8.4	(109.5)	11.3	(125.0)
Density of Timber	0.074	(0.029)	0.528	(0.105)	0.454	(0.101)
Volume per-acre material	-0.002	(0.003)	-0.010	(0.011)	-0.008	(0.010)
Logging costs est. per mmbf	-0.0022	(0.0020)	-0.0036	(0.0053)	-0.0014	(0.0059)
constant	-2.26	(4.39)	72.22	(14.23)	74.48	(13.75)
	N = 699		N = 699		N = 699	
	F(31, 667) = 2.78		F(31, 667) = 51.92		F(31, 667) = 56.35	
	Prob > F = 0.0000		Prob > F = 0.0000		Prob > F = 0.0000	
	R-squared = 0.3008		R-squared = 0.7552		R-squared = 0.7580	

Notes: Species, Forest, and Year dummies included in each specification.

Sample includes Region 6 oral auctions. Species ordered so that sp. 1 is over-estimated.

Table VI: Skewing in Response to Misestimates in Region 5 Sealed Bid Auctions

Dependent Variable	(1) Skew in Right Dir. Dummy=1 if Skew in Right Direction		(2) Skew Δb-Δr		(3) Skew in Right Dir. Dummy=1 if Skew in Right Direction		(4) Skew Δb-Δr		(5) Magnitude of skew = $\lvert\Delta b$-$\Delta r\rvert$ Magnitude of skew	
	Probit		OLS				Fixed Effect Regressions			
	coefficient	s.e.	coefficient	s.e.	coefficient	s.e.	coefficient	s.e.	coefficient	s.e.
Rank Variables										
Rank 2 dummy					0.107	(0.074)	5.181	(8.005)	-11.779	(4.977)
Rank 3 dummy					0.054	(0.074)	-1.468	(8.005)	-17.246	(4.977)
Misestimate Variables										
δ1 = misestimate	5.241	(2.899)	380.929	(240.164)						
Volume and Reserve Price Controls										
Volume est. (mmbf)	-0.213	(0.113)	-37.503	(13.025)						
Average reserve price per mbf	0.008	(0.007)	0.103	(0.491)						
Bidder Participation										
# bidders	0.067	(0.067)	-1.890	(4.165)						
SBA sale (dummy)	-0.389	(0.483)	-19.498	(31.265)						
Other Sale Characteristics										
Contract Length/Volume	-17.706	(11.495)	-971.996	(786.576)						
Density of Timber	-0.004	(0.016)	2.554	(2.456)						
Road costs est. per mmbf	-0.015	(0.023)	-2.191	(2.437)						
constant	2.49	(2.79)	419.68	(262.44)	0.64	(0.05)	-0.09	(5.66)	30.58	(3.52)
	N =63		N = 63		F(55,110) = 2.138		F(38,76) = 2.154		F(38,76) = 5.689	
	Chi-sq(13)	18.63	F(13, 49)	1.4	N=168	n=56	N=117	n=39	N=117	n=39
	Prob>Chi-sq	0.1350	Prob > F	0.1958	corr(u_key, Xb) = -0.0000		corr(u_key, Xb) = -0.0000		corr(u_key, Xb) = -0.0000	
	Psuedo R-sq	0.1564	R-squared	0.4512	F(2, 110) = 1.04		F(2, 76) = 0.38		F(2, 76) = 6.27	
					Prob>F = 0.3572		Prob > F = 0.6846		Prob>F = 0.0030	

Notes: Species, Forest, and Year dummies included in each specification. Sample includes Region 5 sealed bid auctions. Species ordered so that sp. 1 is over-estimated.
Specification (3) includes only sales with at least three bidders.
Specifications (4)-(5) include only sales with at least three bidders and all three bids uncensored.

Table VII: Bidding in Response to Misestimates and Binding Reserve Prices

	(1)		(2)		(3)		(4)		(5)		(6)	
Dependent Variable	**Probability Reserve Prices Do Not Bind**		**ln(Magn. Of Skew)**		**ln(Magn. Of Skew)**		**ln(Magn. Of Skew)**		**ln(Overbid)**		**ln(Overbid)**	
			ln(|Δ*b*-Δ*r*|)		**ln(|Δ*b*-Δ*r*|)**		**ln(|Δ*b*-Δ*r*|)**		**ln((B-R)/Qest)**		**ln((B-R)/Qest)**	
	Full Sample		**Res. doesn't bind**		**Res. doesn't bind**		**Full Sample**		**Res. doesn't bind**		**Res. Binds**	
	Probit		**Heckman Selection Model**		**OLS**		**Censored Normal**		**Heckman Selection Model**			
	coeff.	**Robust s.e.**	**coeff.**	**s.e.**	**coeff.**	**Robust s.e.**	**coeff.**	**Robust s.e.**	**coeff.**	**s.e.**	**coeff.**	**s.e.**
Misestimate Variables (ordered by direction of skew)												
ln(δ1+.3)	-0.061	(0.192)	0.464	(0.251)	0.413	(0.250)	0.573	(0.263)	0.220	(0.152)	0.666	(0.168)
Volume and Reserve Price Controls												
ln(Volume est. (mmbf))	0.435	(0.254)	-0.353	(0.376)	-0.200	(0.436)	-0.539	(0.404)	0.173	(0.233)	0.703	(0.218)
ln(Ave. reserve price per mbf)	-0.109	(0.076)	0.413	(0.114)	0.368	(0.144)	0.396	(0.131)	-0.014	(0.071)	-0.091	(0.066)
Bidder Participation												
SBA sale (dummy)	-0.261	(0.133)	0.274	(0.180)	0.161	(0.182)	0.417	(0.185)	0.075	(0.113)	-0.157	(0.121)
Other Sale Characteristics												
ln(Contract Length/Volume)	0.166	(4.366)	-3.651	(5.009)	-2.734	(4.204)	-2.803	(5.227)	2.211	(3.002)	5.766	(3.933)
ln(Density of Timber)	0.211	(0.047)	-0.035	(0.066)	0.066	(0.075)	-0.137	(0.074)	0.107	(0.048)	0.271	(0.042)
ln(Volume per-acre material)	0.084	(0.027)	-0.055	(0.040)	-0.014	(0.046)	-0.086	(0.045)	0.015	(0.027)	0.099	(0.024)
ln(Log. costs est. per mmbf)	-0.616	(0.248)	0.588	(0.366)	0.361	(0.435)	0.849	(0.413)	-0.105	(0.232)	-0.724	(0.218)
Binding Reserve Prices												
ln(% of est. volume on sp. 1)	-1.061	(0.216)										
Distributional Parameters												
Rho (correlation across eqn's)			-0.587						0.167		-0.990	
Sigma (std. dev. of bid or skew eqn)			1.440						0.838		0.937	
Lambda (Rho*Sigma)			-0.845	(0.028)					0.140	(0.218)	-0.928	(0.034)
Std. Dev (cens. Normal model)							1.718	(0.093)				
constant	2.4	(1.4)	0.7	(2.1)	1.1	(2.4)	0.4	(2.3)	4.9	(1.3)	10.1	(1.2)
	N =694		N =694		N = 408		N = 694		N = 694		N = 694	
	Chi-sq(17)	134.9	Chi-sq(34)	162.13	F(16, 391)	4.79	Chisq(16)	118.38	Chisq(34)	151.35	Chisq(34)	-101.35
	Pr > Chisq	0.0000	Pr > Chisq	0.0000	Prob > F	0.0000	Pr > Chisq	0.0000	Pr > Chisq	0.0000	Pr > Chisq	0.0000
	Ps. R-sq	0.1434			R-sq	0.1357						

Notes: Species, Forest, and Year dummies included in each specification. Sample includes Region 6 oral auctions where skew is strictly positive.
Species ordered so that the winner's skew is positive.
Columns (2) and (5) are estimated using the full sample to determine the probability that the reserve price binds, but the skewing and bidding equations are estimated on the subsample where the reserve prices do not bind. Column (6) estimates the bidding equation when the reserve prices do bind.
Column (3) includes only sales where reserve prices do not bind.